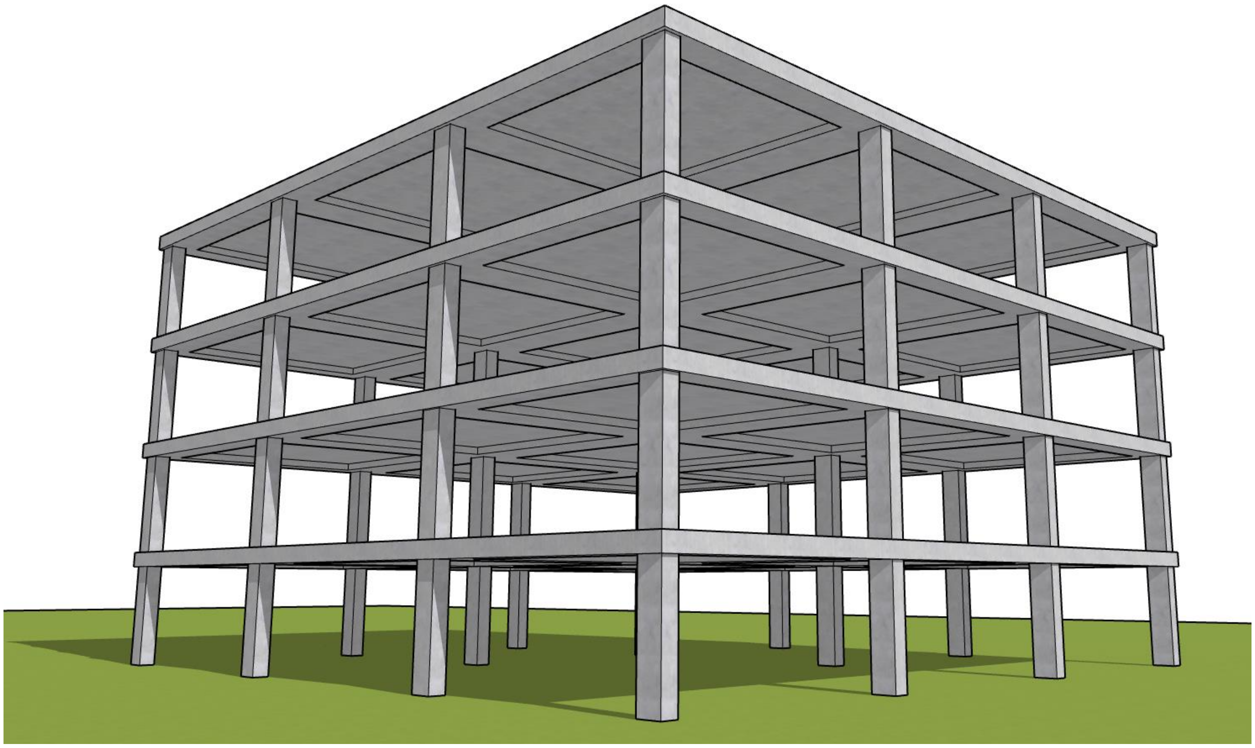
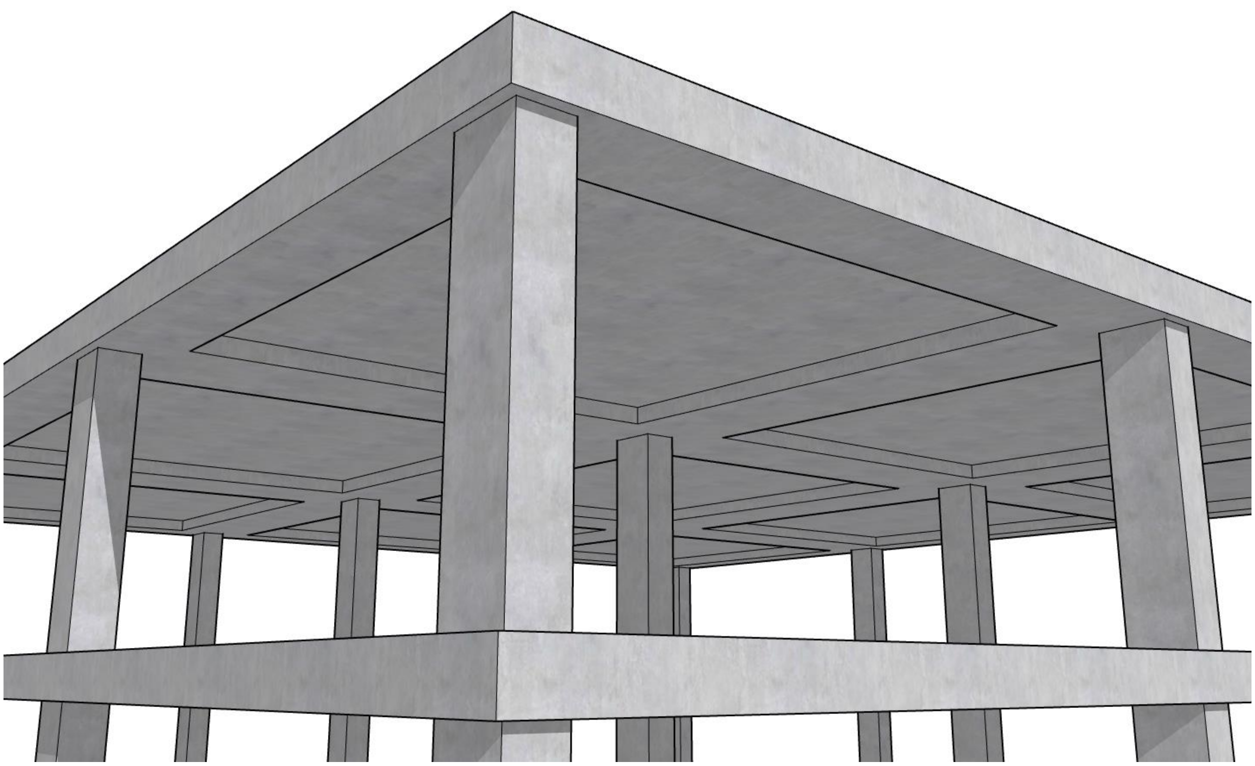
Two-Way Slab with Beams Design and Detailing (CAC Design Handbook)





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The concrete floor slab system shown below is for an intermediate floor to be designed considering superimposed dead load = 1.6 kN/m^2, and unfactored live load = 4.8 kN/m^2. The lateral loads are independently resisted by shear walls. The use of flat plate system will be checked. If the use of flat plate is not adequate, the use of slab system with beams between all supports will be investigated. The analysis procedure “Elastic Frame Method (EFM)” prescribed in ***CSA A23.3-14*** is illustrated in detail in this example (Example #4 from the CAC Design Handbook). The hand solution from EFM is also used for a comparison with the Reference results using Direct Design Method (DDM) and results of the engineering software program spSlab. Explanation of the EFM is available in StructurePoint Video Tutorials page.

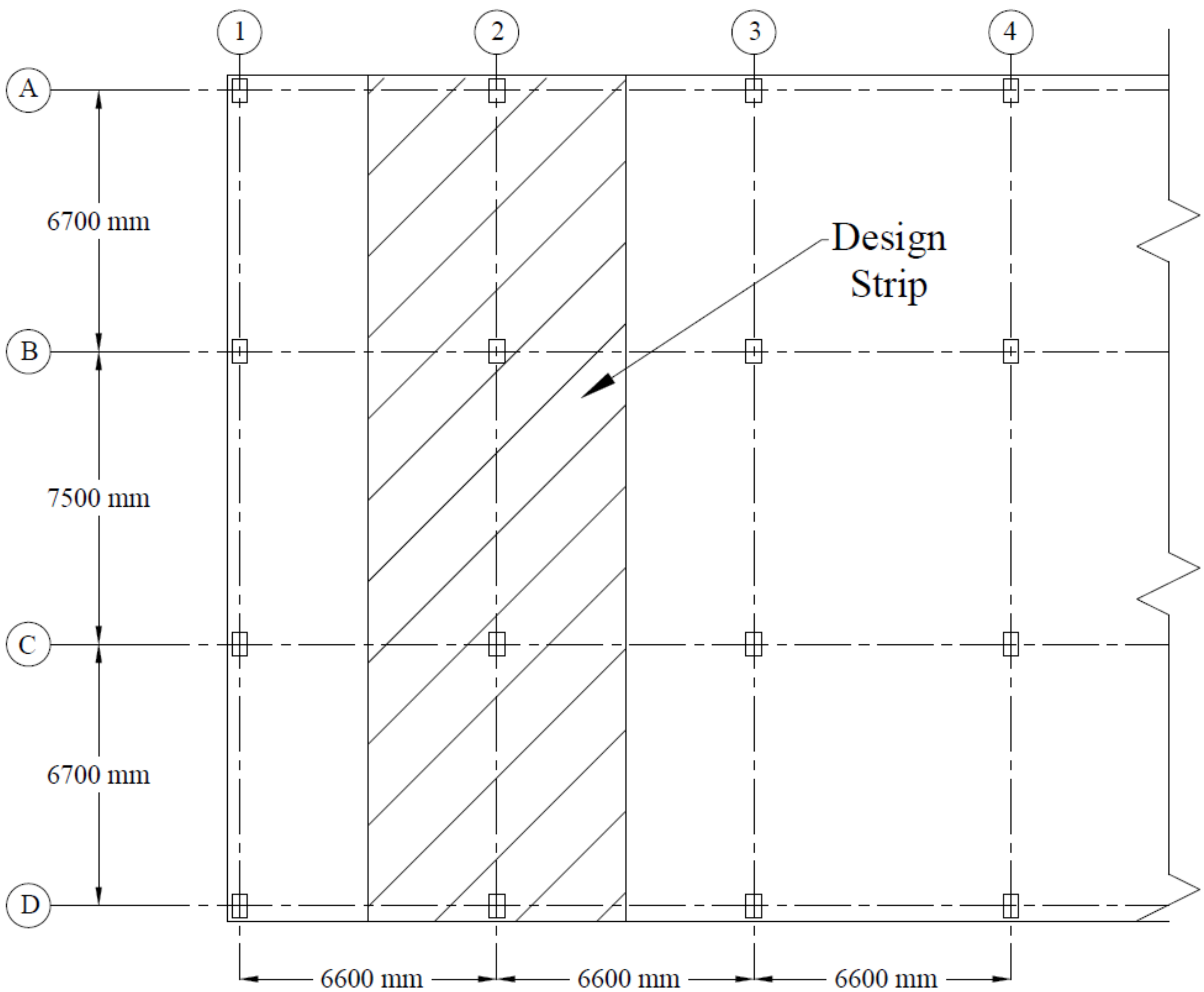


Figure 1 - Two-Way Flat Concrete Floor System

Contents

Code

Design of Concrete Structures (CSA A23.3-14)

Reference

CAC Concrete Design Handbook, 4th Edition, Cement Association of Canada

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association

Design Data

Floor-to-Floor Height = 3 m (provided by architectural drawings)

Superimposed Dead Load, SDL $=1.6$ kN/m^2

Live Load, LL = 4.8 kN/m^2

$f'_c = 25$ MPa (for slabs)

$f'_c = 25$ MPa (for columns)

$f'_y = 400$ MPa

Column Dimensions = 400 mm x 600 mm

Solution

1. Preliminary Member Sizing

<u>For slab without beams (flat plate)</u>

a) <u>Slab minimum thickness - Deflection</u>

<u>CSA A23.3-14 (13.2)</u>

Minimum member thickness and depths from CSA A23.3-14 will be used for preliminary sizing.

Using CSA A23.3-14 minimum slab thickness for two-way construction without interior beams in ***Section 13.2.3.***

<u>Exterior Panels (E-W Direction Governs):</u>

$$h_{s,\min} = 1.1 \times \frac{l_n\left(0.6 + f_y / 1000\right)}{30} = 1.1 \times \frac{6200\left(0.6 + 400/1000\right)}{30} = 227 \text{ mm}$$ ***<u>CSA A23.3-14 (13.2.3)</u>***

But not less than 120 mm. ***<u>CSA A23.3-14 (13.2.1)</u>***

Where l_n = length of clear span in the short direction = 6600 – 400 = 6200 mm

Interior Panels (N-S Direction Governs):

$$h_{s,\min} = \frac{l_n\left(0.6 + f_y / 1000\right)}{30} = \frac{6900\left(0.6 + 400/1000\right)}{30} = 230 \text{ mm}$$ **CSA A23.3-14 (13.2.3)**

But not less than 120 mm. **CSA A23.3-14 (13.2.1)**

Where l_n = length of clear span in the long direction = 7500 – 600 = 6900 mm

Try 250 mm slab for all panels (self-weight = 5.89 kN/m^2)

b) Slab one way shear strength

Evaluate the average effective depth (Figure 2):

$$d_t = t_{slab} - c_{clear} - d_b - \frac{d_b}{2} = 250 - 25 - 16 - \frac{16}{2} = 201 \text{ mm}$$

$$d_l = t_{slab} - c_{clear} - \frac{d_b}{2} = 250 - 25 - \frac{16}{2} = 217 \text{ mm}$$

$$d_{avg} = \frac{d_l + d_t}{2} = \frac{201 + 217}{2} = 209 \text{ mm}$$

Where:

c_{clear} = 20 mm for 15M steel bar **CSA A23.3-14 (Annex A. Table 17)**

Note that the reference used 25 mm as clear cover, in this example the clear cover used is 25 mm to be consistent with reference.

d_b = 16 mm for 15M steel bar

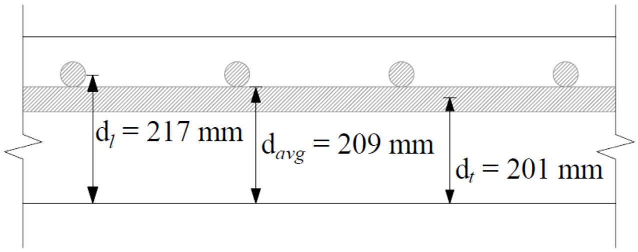


Figure 2 - Two-Way Flat Concrete Floor System

Load Combination 1:

Factored dead load, $w_{df} = 1.4 \times (5.89 + 1.6) = 10.49 \text{ kN/m}^2$ **CSA A23.3-14 (Annex C. Table C.1 a)**

Total factored load $w_f = 10.49 \text{ kN/m}^2$

Load Combination 2:

Factored dead load, $w_{df} = 1.25 \times (5.89 + 1.6) = 9.36 \text{ kN/m}^2$

Factored live load, $w_{lf} = 1.5 \times 4.8 = 7.20 \text{ kN/m}^2$ ***CSA A23.3-14 (Annex C. Table C.1 a)***

Total factored load $w_f = w_{df} + w_{lf} = 16.56 \text{ kN/m}^2$ (Controls)

Check the adequacy of slab thickness for beam action (one-way shear) ***CSA A23.3-14 (13.3.6)***

At an interior column:

The critical section for one-way shear is extending in a plane across the entire width and located at a distance, d_v from the face of support or concentrated load (see Figure 3). ***CSA A23.3-14 (13.3.6.1)***

Consider a 1 m. wide strip.

Tributary area for one-way shear is $A_{Tributary} = \left(\dfrac{\left[\left(\dfrac{7500}{2} \right) - \left(\dfrac{600}{2} \right) - 188 \right] \times (1000)}{1000^2} \right) = 3.26 \text{ m}^2$

$$V_f = w_f \times A_{Tributary} = 16.56 \times 3.26 = 54.03 \text{ kN}$$

$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$ ***CSA A23.3-14 (Eq. 11.6)***

Where:

$\lambda = 1$ for normal weight concrete ***CSA A23.3-14 (8.6.5)***

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm ***CSA A23.3-14 (11.3.6.2)***

$d_v = \text{Max } (0.9 d_{avg}, 0.72h) = \text{Max } (0.9 \times 209, 0.72 \times 250) = \text{Max } (188, 180) = 188 \text{ mm}$ ***CSA A23.3-14 (3.2)***

$\sqrt{f'_c} = 5 \text{ MPa} < 8 \text{ MPa}$ ***CSA A23.3-14 (11.3.4)***

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{25} \times 1000 \times \frac{188}{1000} = 128.3 \text{ kN} > V_f$$

Slab thickness of 250 mm is adequate for one-way shear.

c) Slab two-way shear strength

Check the adequacy of slab thickness for punching shear (two-way shear) at an interior column (Figure 4):

Shear prerimeter: $b_0 = 2 \times (600 + 400 + 2 \times 209) = 2836 \text{ mm}$ ***CSA A23.3-14 (13.3.3)***

Tributary area for two-way shear is

$$A_{Tributary} = \left(\frac{7.5 + 6.7}{2} \times 6.6 \right) - \left(\frac{600 + 209}{1,000} \times \frac{400 + 209}{1,000} \right) = 46.86 - 0.49 = 46.37 \text{ m}^2$$

The factored resisiting shear stress, V_r shall be the smallest of : ***CSA A23.3-14 (13.3.4.1)***

1. $v_r = v_c = \left(1 + \frac{2}{\beta_c}\right) 0.19 \lambda \phi_c \sqrt{f'_c}$ **CSA A23.3-14 (Eq. 13.5)**

$$v_r = \left(1 + \frac{2}{1.5}\right) \times 0.19 \times 0.65 \times \sqrt{25} = 1.44 \text{ MPa}$$

Where $\beta_c = \frac{600}{400} = 1.5$ (ratio of long side to short side of the column) **CSA A23.3-14 (13.3.4.1)**

2. $v_r = v_c = \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f'_c}$ **CSA A23.3-14 (Eq. 13.6)**

$$v_r = \left(\frac{4 \times 209}{2836} + 0.19\right) \times 1 \times 0.65 \times \sqrt{25} = 1.58 \text{ MPa}$$

3. $v_r = v_c = 0.38 \lambda \phi_c \sqrt{f'_c} = 0.38 \times 1 \times 0.65 \times \sqrt{25} = 1.24 \text{ MPa}$ **CSA A23.3-14 (Eq. 13.7)**

$$v_{f,ave} = \frac{V_f}{b_o d} = \frac{16.56 \times \left(\frac{7.5 + 6.7}{2} \times 6.6\right)}{2836 \times 209} \times 1{,}000 = 1.309 \text{ MPa}$$

$\frac{v_r}{v_{f,ave}} = \frac{1.240}{1.309} = 0.94 < 1.20$ ***CAC Concrete Design Handbook 4th Edition (5.2.3)***

Slab thickness of 250 mm is not adequate for two-way shear.

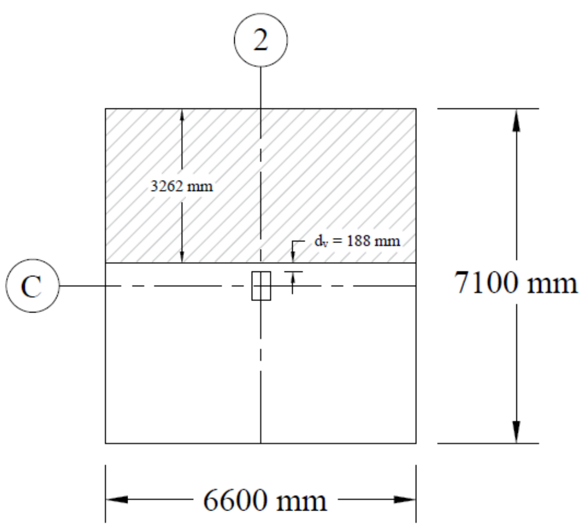


Figure 3 - Critical Section for One-Way

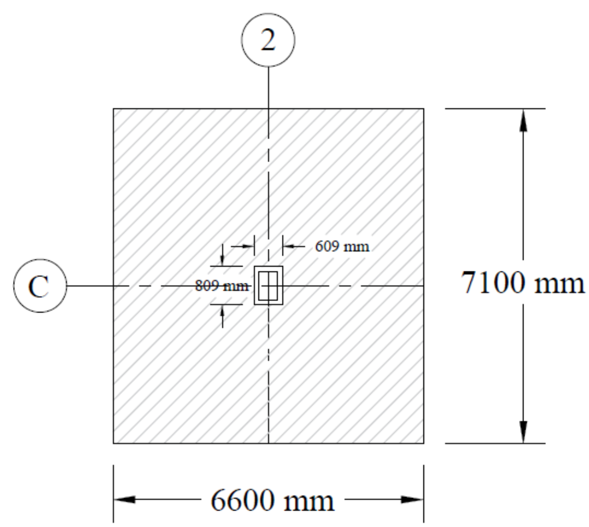


Figure 4 - Critical Section for Two-Way

For slab with beams

a) Slab minimum thickness – Deflection

Control of deflections. ***CSA A23.3-14 (13.2.5)***

In lieu of detailed calculation for deflections, CSA A23.3 Code gives minimum thickness for two-way slab with beams between all supports on all sides in ***Clause 13.2.5***.

Ratio of moment of inertia of beam section to moment of inertia of a slab (α) is computed as follows:

$$\alpha = \frac{I_b}{I_s}$$ ***CSA A23.3 (13.2.5)***

The moment of inertia for the effective beam and slab sections can be calculated as follows:

$$I_b = \frac{b_w h^3}{12}\left(2.5\left(1-\frac{h_s}{h}\right)\right)$$ ***CSA A23.3 (Eq. 13.4)***

The preliminary values are shown below and will be checked in next steps:

Slab thickness (h_s) = 200 mm

Slab width (b) = 6600 mm for interior and 3300 mm exterior (North-South)

= 7100 mm for interior and 3350 mm exterior (East-West)

Beam depth (h) = 400 mm

Beam width (b_w) = 1400 mm for interior and 800 mm exterior

Edge Beams:

The effective beam and slab sections for the computation of stiffness ratio for edge beam is calculated as follows:

For North-South Edge Beams:

$$I_b = \frac{800\times 400^3}{12}\left(2.5\times\left(1-\frac{200}{400}\right)\right) = 5.33\times 10^9 \text{ mm}^4$$

$$I_s = \frac{3300\times 200^3}{12} = 2.20\times 10^9 \text{ mm}^4$$

$$\alpha = \frac{5.33\times 10^9}{2.20\times 10^9} = 2.42$$

For East-West Edge Beams:

$$I_b = \frac{800\times 400^3}{12}\left(2.5\times\left(1-\frac{200}{400}\right)\right) = 5.33\times 10^9 \text{ mm}^4$$

$$I_s = \frac{3350\times 200^3}{12} = 2.23\times 10^9 \text{ mm}^4$$

$$\alpha = \frac{5.33\times 10^9}{2.23\times 10^9} = 2.39$$

Interior Beams:

For North-South Interior Beams:

$$I_b = \frac{1400\times400^3}{12}\left(2.5\times\left(1-\frac{200}{400}\right)\right) = 9.33\times10^9 \text{ mm}^4$$

$$I_s = \frac{6600\times200^3}{12} = 4.40\times10^9 \text{ mm}^4$$

$$\alpha = \frac{9.33\times10^9}{4.40\times10^9} = 2.12$$

For East-West Interior Beams:

$$I_b = \frac{1400\times400^3}{12}\left(2.5\times\left(1-\frac{200}{400}\right)\right) = 9.33\times10^9 \text{ mm}^4$$

$$I_s = \frac{7100\times200^3}{12} = 4.73\times10^9 \text{ mm}^4$$

$$\alpha = \frac{9.33\times10^9}{4.73\times10^9} = 1.97$$

The average of α for the beams on four sides of exterior and interior panels are calculated as:

For exterior panels: $\alpha_m = \frac{(2.42+2.39+2.12+1.97)}{4} = 2.23$

For interior panels: $\alpha_m = \frac{(2\times2.12+2\times1.97)}{4} = 2.05$

α_m shall not be taken greater than 2.0, then α_m = 2.0 for both exterior and interior panels.

The minimum slab thickness is given by:

$$h_{\min} = \frac{l_n\left(0.6+\frac{f_y}{1,000}\right)}{30+4\beta\alpha_m}$$ **CSA A23.3-14 (13.2.5)**

Where:

l_n = clear span in the long direction measured face to face of columns = 6.9 m = 6900 mm

$$\beta = \frac{\text{clear span in the long direction}}{\text{clear span in the short direction}} = \frac{7500-600}{6600-400} = 1.113$$

$$h_{\min} = \frac{6900\left(0.6+\frac{400}{1000}\right)}{30+4\times1.113\times2} = 177.4 \text{ mm}$$

The assumed thickness is more than the h_{min}. Use 200 mm slab thickness.

2. Two-Way Slab Analysis and Design

CSA A23.3 states that a regular slab system may be designed using any procedure satisfying conditions of equilibrium and compatibility with the supports, provided that it is shown that the factored resistance at every section is at least equal to the effects of the factored loads and that all serviceability conditions, including specified limits on deflections, are met. ***CSA A23.3-14 (13.5.1)***

CSA A23.3 permits the use of Plastic Plate Theory Method (PPTM), Theorems of Plasticity Method (TPM), Direct Design Method (DDM) and Elastic Frame Method (EFM); known as Equivalent Frame Method in the ACI; for the gravity load analysis of orthogonal frames. The following sections outline a brief description of DDM, a detailed hand solution using EFM and an automated solution using spSlab software respectively.

2.1. Direct Design Method (DDM)

Two-way slabs satisfying the limits in ***CSA A23.3-14 (13.9)*** are permitted to be designed in accordance with the DDM.

2.1.1. Direct design method limitations

There shall be a minimum of three continuous spans in each direction (3 spans) ***CSA A23.3-14 (13.9.1.2)***

Successive span lengths centre-to-centre of supports in each direction shall not differ by more than one- third of the longer span ((7500-6700)/6700 = 0.12 < 0.33) ***CSA A23.3-14 (13.9.1.3)***

All loads shall be due to gravity only and uniformly distributed over an entire panel (Loads are uniformly distributed over the entire panel) ***CSA A23.3-14 (13.9.1.4)***

The factored live load shall not exceed twice the factored dead load (Service live-to-dead load ratio of (4.8/(24*200/1000) = 1.00 < 2.0) ***CSA A23.3-14 (13.9.1.4)***

Since all the criteria are met, Direct Design Method can be utilized.

Detailed illustration of analysis and design of two-way slab using DDM can be found in "Two-Way Flat Plate Concrete Slab Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website. This example focuses on the analysis of two-way slab with beams using EFM.

2.1.2. Design moments

a. Calculate the total factored static moment:

$$M_o = \frac{w_f \ell_{2a} \ell_n^{\,2}}{8}$$ ***CSA A23.3-14 (13.9.1.4)***

Distribute the total factored moment, M_o, in an interior and end span: ***CSA A23.3-14 (13.9.3.1 &13.9.3.2)***

Table 1 - Distribution of M_o along the span		
Location		**Total Design Strip Moment, M_{DES} (kN.m)**
Exterior Span	Exterior Negative	$0.26 \times M_o = 34.8$
	Positive	$0.52 \times M_o = 69.6$
	Interior Negative	$0.70 \times M_o = 93.68$
Interior Span	Positive	$0.35 \times M_o = 46.8$

b. Calculate the column strip moments. ***CSA A23.3-14 (13.11.2)***

That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips. ***CSA A23.3-14 (13.11.3.1)***

Table 2 - Lateral Distribution of the Total Design Strip Moment, M_{DES}				
Location		**Total Design Strip Moment, (kN.m)**	**Column Strip Moment, (kN.m)**	**Moment in Two Half Middle Strips, (kN.m)**
Exterior Span	Exterior Negative*	34.8	$1.00 \times M_{DES} = 34.8$	$0.00 \times M_{DES} = 0.0$
	Positive	69.6	$0.6 \times M_{DES} = 41.8$	$0.4 \times M_{DES} = 27.8$
	Interior Negative*	93.68	$0.8 \times M_{DES} = 74.94$	$0.2 \times M_{DES} = 18.7$
Interior Span	Positive	46.8	$0.6 \times M_{DES} = 28.1$	$0.4 \times M_{DES} = 18.7$
* All negative moments are at face of support.				

Figure 5 – Sample Calculations Using DDM from “Two-Way Flat Plate Concrete Slab Floor Analysis and Design” Design Example

2.2. Elastic Frame Method (EFM)

EFM (also known as Equivalent Frame Method in the ACI 318) is the most comprehensive and detailed procedure provided by the CSA A23.3 for the analysis and design of two-way slab systems where these systems may, for purposes of analysis, be considered a series of plane frames acting longitudinally and transversely through the building. Each frame shall be composed of equivalent line members intersecting at member centrelines, shall follow a column line, and shall include the portion of slab bounded laterally by the centreline of the panel on each side. ***CSA A23.3-14 (13.8.1.1)***

Probably the most frequently used method to determine design moments in regular two-way slab systems is to consider the slab as a series of two-dimensonal frames that are analyzed elastically. When using this analogy, it is essential that stiffness properties of the elements of the frame be selected to properly represent the behavior of the three-dimensional slab system.

In a typical frame analysis it is assumed that at a beam-column cconnection all members meeting at the joint undergo the same rotaion. For uniform gravity loading this reduced restraint is accounted for by reducing the effective stiffness of the column by either Clause 13.8.2 or Clause 13.8.3. ***CSA A23.3-14 (N.13.8)***

Each floor and roof slab with attached columns may be analyzed separately, with the far ends of the columns considered fixed. ***CSA A23.3-14 (13.8.1.2)***

The moment of inertia of column and slab-beam elements at any cross-section outside of joints or column capitals shall be based on the gross area of concrete at that section. ***CSA A23.3-14 (13.8.2.5)***

An equivalent column shall be assumed to consist of the actual columns above and below the slab-beam plus an attached torsional member transverse to the direction of the span for which moments are being determined.
CSA A23.3-14 (13.8.2.5)

2.2.1. Elastic frame method limitations

In EFM, live load shall be arranged in accordance with 13.8.4 which requires:

- slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***CSA A23.3-14 (13.8.4)***
- Complete analysis must include representative interior and exterior equivalent elastic frames in both the longitudinal and transverse directions of the floor. ***CSA A23.3-14 (13.8.1.1)***
- Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***CSA A23.3-14 (3.1a)***
- For slab systems with beams between sypports, the relative effective stiffness of beams in the two directions is not less than 0.2 or greater than 5.0. ***CSA A23.3-14 (3.1b)***
- Column offsets are not greater than 20% of the span (in the direction of offset) from either axis between centerlines of successive columns. ***CSA A23.3-14 (3.1c)***

The reinforcement is placed in an orthogonal grid. ***CSA A23.3-14 (3.1d)***

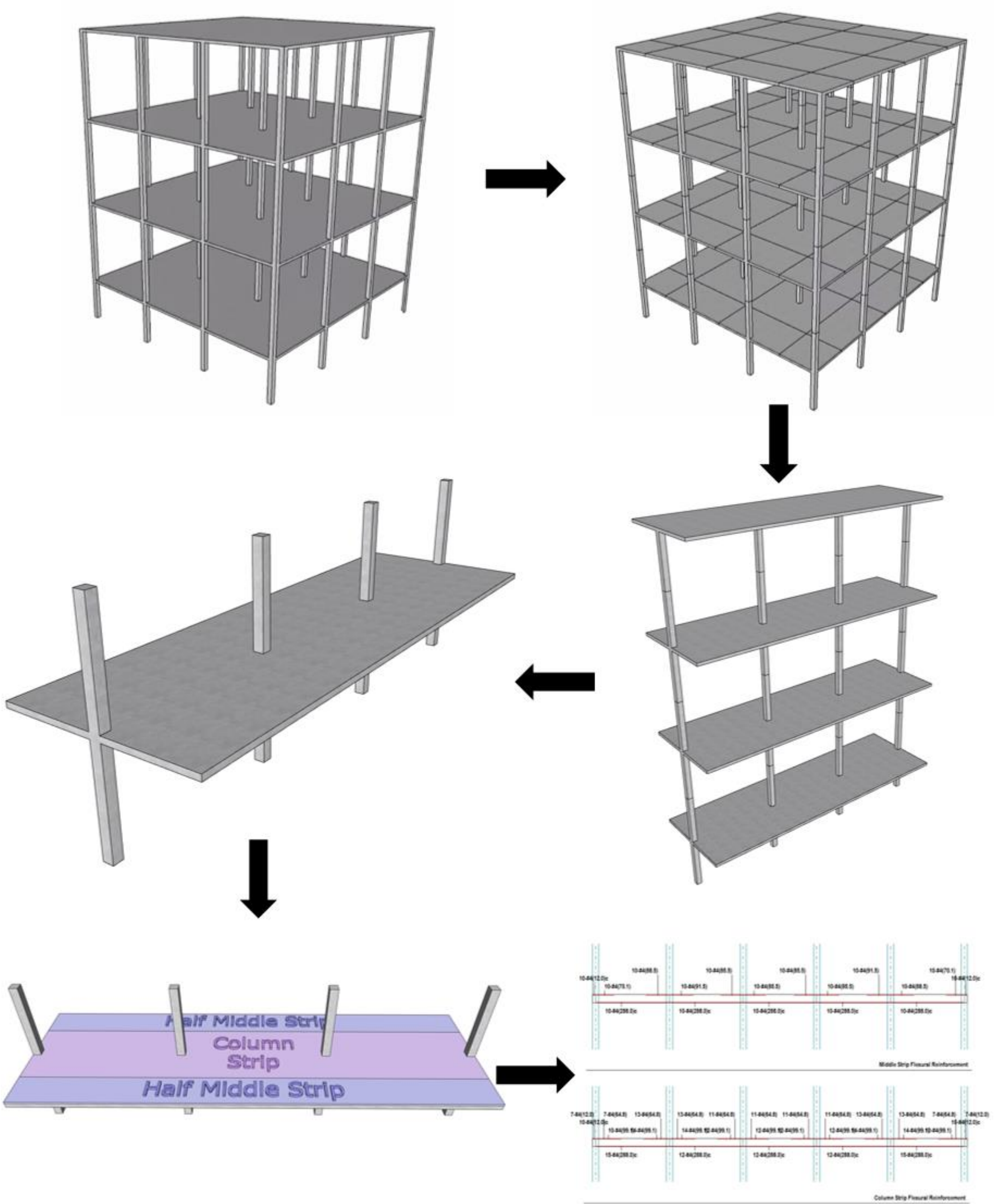


Figure 6 – Elastic (Equivalent) Frame Methodology

2.2.2. Frame members of elastic frame

Determine moment distribution factors and fixed-end moments for the elastic frame members. The moment distribution procedure will be used to analyze the equivalent frame. Stiffness factors k, carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a. Flexural stiffness of slab-beams at both ends, K_{sb}

For Interior Span:

$$\frac{c_{N1}}{\ell_1} = \frac{600}{7500} = 0.080 \;,\quad \frac{c_{N2}}{\ell_2} = \frac{400}{6600} = 0.061$$

For $c_{F1} = c_{N2}$, stiffness factors, $k_{NF} = k_{FN} = 4.09$ ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb} = k_{NF}\dfrac{E_{cs}I_s}{\ell_1} = 4.09\dfrac{E_{cs}I_s}{\ell_1}$ ***PCA Notes on ACI 318-11 (Table A1)***

Where I_{sb} is the moment of inertia of slab-beam section shown in Figure 7 and can be computed as follows:

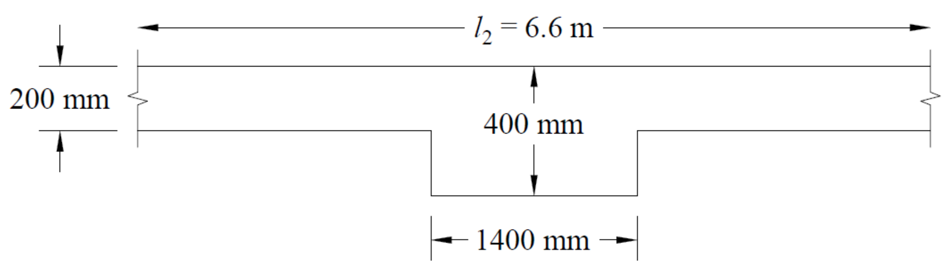


Figure 7 – Cross-Section of Slab-Beam

$$C_t = 1 + (A-1)B^3 + \frac{3(1-B)^2 B(A-1)}{1+B(A-1)} = 1.95$$ ***PCA Notes on ACI 318-11 (Figure 20-21)***

Where A = b/b_w = 6600 / 1400 = 4.71 and B = h_s/h = 200 / 400 = 0.5

$$I_s = C_t\left(\frac{b_w h^3}{12}\right) = 1.95\left(\frac{1400\times 400^3}{12}\right) = 14.57\times 10^9 \text{ mm}^4$$ ***PCA Notes on ACI 318-11 (Figure 20-21)***

$$E_{cs} = \left(3300\sqrt{f_c'} + 6900\right)\left(\frac{\gamma_c}{2300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2)***

$$E_{cs} = (3300\sqrt{25} + 6900)\left(\frac{2402.8}{2300}\right)^{1.5} = 24{,}986 \text{ MPa}$$

$$K_{sb} = 4.09\times 24{,}986\times\frac{14.57\times 10^9}{7500}\times 10^{-3} = 198.6\times 10^6 \text{ N.m}$$

Carry-over factor COF = 0.50 ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment $\text{FEM} = 0.0843 w_u \ell_2 \ell_1^2$ ***PCA Notes on ACI 318-11 (Table A1)***

For Exterior Span:

$$\frac{c_{N1}}{\ell_1}=\frac{600}{6700}=0.090\ ,\ \frac{c_{N2}}{\ell_2}=\frac{600}{6600}=0.061$$

For $c_{F1}=c_{N2}$, stiffness factors, $k_{NF}=k_{FN}=4.10$ ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb}=k_{NF}\frac{E_{cs}I_s}{\ell_1}=4.10\frac{E_{cs}I_s}{\ell_1}$ ***PCA Notes on ACI 318-11 (Table A1)***

$$K_{sb}=4.10\times 24{,}986\times\frac{14.57\times10^9}{6700}\times10^{-3}=222.8\times10^6\text{ N.m}$$

Carry-over factor COF = 0.51 ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment $\text{FEM}=0.0843w_u\ell_2\ell_1^{\ 2}$ ***PCA Notes on ACI 318-11 (Table A1)***

b. Flexural stiffness of column members at both ends, K_c

Referring to ***Table A7, Appendix 20A***:

$t_a=400-200/2=300$ mm , $t_b=200/2=100$ mm

$H=3.0\text{ m}=3000$ mm, $H_c=3000-400=2600$ mm

$$\frac{t_a}{t_b}=3.00,\ \frac{t_b}{t_a}=0.33,\ \frac{H}{H_c}=1.15$$

Thus, $k_{c,\ top}=6.33$ and $k_{c,\ bottom}=5.13$ by interpolation.

$$I_c=\frac{b\times h^3}{12}=\frac{400(600)^3}{12}=7.20\times10^9\ \text{mm}^4$$

$$E_{cc}=(3{,}300\sqrt{f_c^{'}}+6{,}900)\left(\frac{\gamma_c}{2{,}300}\right)^{1.5}$$ ***CSA A23.3-14(8.6.2.2)***

$$E_{cc}=(3300\sqrt{25}+6900)\left(\frac{2402.8}{2300}\right)^{1.5}=24{,}986\text{ MPa}$$

$\ell_c=3.0\text{ m}=3000$ mm

$$K_c=\frac{k_cE_{cc}I_c}{\ell_c}$$ ***PCA Notes on ACI 318-11 (Table A7)***

$$K_{c,top}=6.33\times24986\times\frac{7.20\times10^9}{3000}\times10^{-3}=380\times10^6\ \text{N.m}$$

$$K_{c,bottom}=5.13\times24986\times\frac{7.20\times10^9}{3000}\times10^{-3}=308\times10^6\ \text{N.m}$$

c. Torsional stiffness of torsional members, K_t

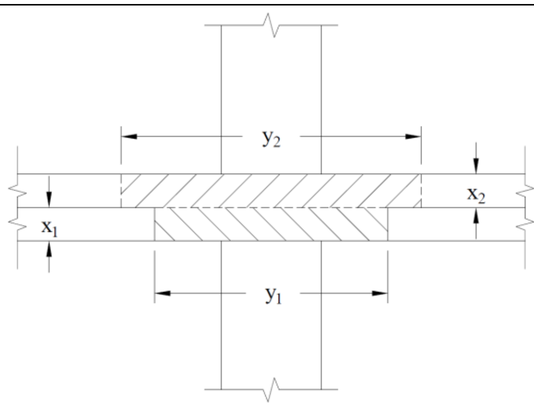
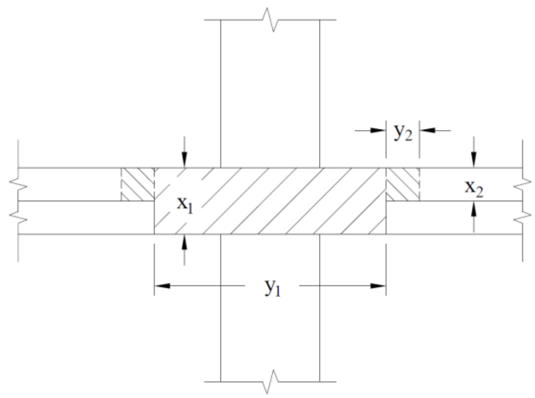
$$K_t = \sum \frac{9E_{cs}C}{\left[\ell_t\left(1-\frac{c_2}{\ell_t}\right)^3\right]}$$ **_CSA A23.3-14 (13.8.2.8)_**

For Interior Columns:

$$K_{t\_int} = \frac{9\times 24{,}986\times 24.90\times 10^9}{6600\times\left(1-\frac{400}{6600}\right)^3}\times 10^{-3} = 102.3\times 10^7\ \text{N.m}$$

Where:

$$C = \sum\left(1-0.63\frac{x}{y}\right)\left(\frac{x^3y}{3}\right)$$ **_CSA A23.3-14 (13.8.2.9)_**

<table>
<tr><td>x_1 = 200 mm
y_1 = 1400 mm
$C_1 = 3.40\times10^9$</td><td>x_2 = 200 mm
y_2 = 1800 mm
$C_2 = 4.46\times10^9$</td><td>x_1 = 400 mm
y_1 = 1400 mm
$C_1 = 24.49\times10^9$</td><td>x_2 = 200 mm
y_2 = 200 mm
$C_2 = 0.20\times10^9$</td></tr>
<tr><td colspan="2">$\sum C = 3.40\times10^9 + 4.46\times10^9 = 7.90\times10^9$ mm^4</td><td colspan="2">$\sum C = 24.49\times10^9 + 2\times0.20\times10^9 = 24.90\times10^9$ mm^4</td></tr>
<tr><td colspan="2">
</td><td colspan="2">
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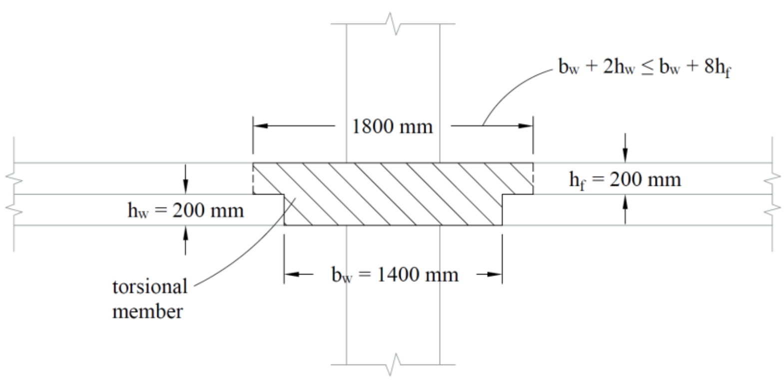


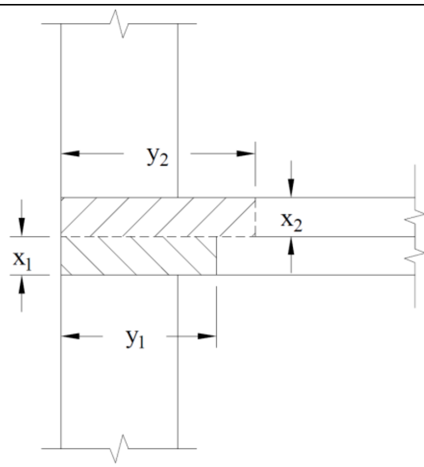
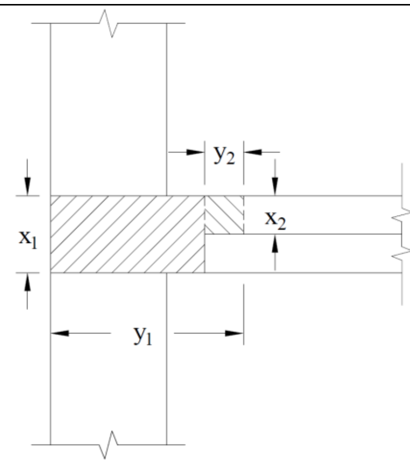
Figure 8 – Attached Torsional Member at Interior Column

For Exterior Columns:

$$K_{t\_ext} = \frac{9\times 24{,}986\times 11.9\times 10^{9}}{6600\times\left(1-\dfrac{400}{6600}\right)^{3}}\times 10^{-3} = 48.86\times 10^{7}\ \text{N.m}$$

Where:

$$C = \sum\left(1-0.63\frac{x}{y}\right)\left(\frac{x^{3}y}{3}\right)$$ ***CSA A23.3-14 (13.8.2.9)***

x_1 = 200 mm y_1 = 800 mm $C_1 = 1.80\times10^9$	x_2 = 200 mm y_2 = 1000 mm $C_2 = 2.33\times10^9$	x_1 = 400 mm y_1 = 800 mm $C_1 = 11.69\times10^9$	x_2 = 200 mm y_2 = 200 mm $C_2 = 0.20\times10^9$
$\sum C = 1.80\times10^9 + 2.33\times10^9 = 4.10\times10^9$ mm^4		$\sum C = 11.69\times10^9 + 0.20\times10^9 = 11.90\times10^9$ mm^4	
			

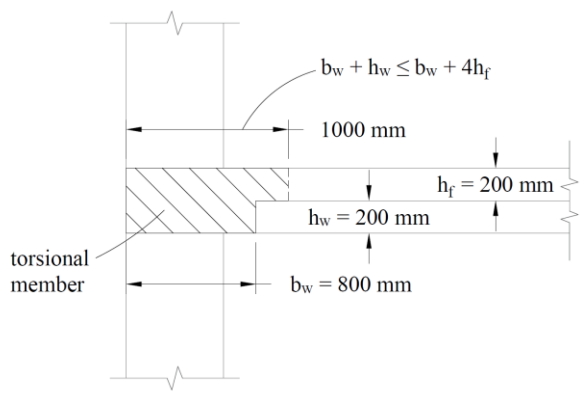


Figure 9 – Attached Torsional Member at Exterior Column

d. Increased torsional stiffness due to parallel beams, K_{ta}.

For Interior Columns:

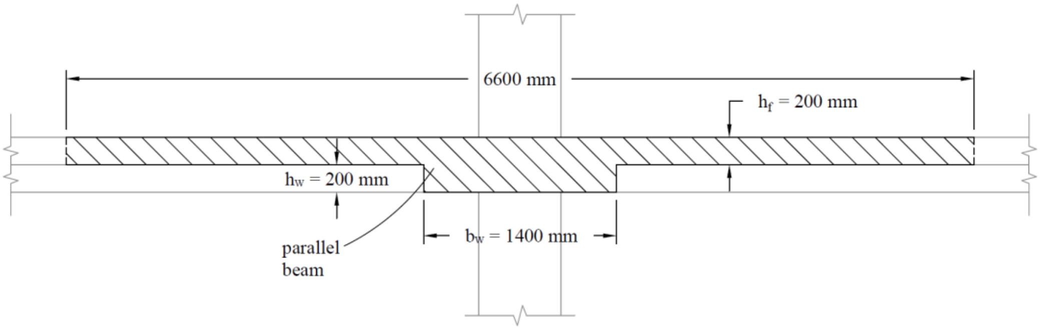


Figure 10 – Slab-Beam in the Direction of Analysis

$$K_{ta\_int} = \frac{K_{t\_int} I_{sb}}{I_s} = \left(1.00\times10^9\right)\times\frac{14.6\times10^9}{4.40\times10^9} = 3.40\times10^9 \text{ N.m}$$

Where:

$$I_s = \frac{l_2 \times h^3}{12} = \frac{6600\times200^3}{12} = 4.40\times10^9 \text{ mm}^4$$

For Exterior Columns:

$$K_{ta\_ext} = \frac{K_{t\_ext} I_{sb}}{I_s} = \left(0.49\times10^9\right)\times\frac{14.6\times10^9}{4.40\times10^9} = 1.60\times10^9 \text{ N.m}$$

e. Equivalent column stiffness, K_{ec}

$$K_{ec} = \frac{\sum K_c \times \sum K_{ta}}{\sum K_c + \sum K_{ta}}$$

Where $\sum K_{ta}$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

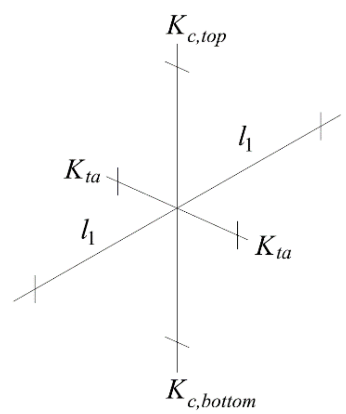


Figure 11 – Equivalent Column Stiffness

For Interior Columns:

$$K_{ec\_int} = \frac{(379.6\times10^6 + 307.6\times10^6)(2\times3.4\times10^9)}{(379.6\times10^6 + 307.6\times10^6) + (2\times3.4\times10^9)} = 623.9\times10^6 \text{ N.m}$$

For Exterior Columns:

$$K_{ec\_ext} = \frac{(379.6\times10^6 + 307.6\times10^6)(2\times1.6\times10^9)}{(379.6\times10^6 + 307.6\times10^6) + (2\times1.6\times10^9)} = 566.9\times10^6 \text{ N.m}$$

f. Slab-beam joint distribution factors, *DF*

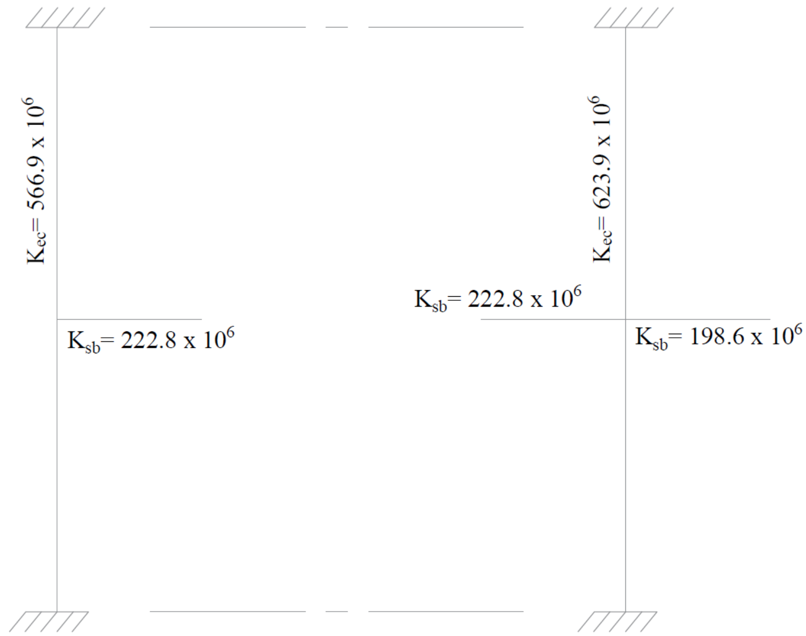


Figure 12 – Slab and Column Siffness

At exterior joint:

$$DF = \frac{222.8\times10^6}{\left(222.8\times10^6 + 566.9\times10^6\right)} = 0.282$$

At interior joint:

$$DF_{Ext} = \frac{222.8\times10^6}{\left(222.8\times10^6 + 198.6\times10^6 + 623.9\times10^6\right)} = 0.213$$

$$DF_{Int} = \frac{198.6\times10^6}{\left(222.8\times10^6 + 198.6\times10^6 + 623.9\times10^6\right)} = 0.190$$

COF for slab-beam = 0.50 for Interior Span

= 0.51 for Exterior Span

2.2.3. Elastic frame analysis

Determine negative and positive moments for the slab-beams using the moment distribution method. Since the unfactored live load does not exceed three-quarters of the unfactored dead load, design moments are assumed to occur at all critical sections with full factored live on all spans. **CSA A23.3-14 (13.8.4.2)**

$$\frac{L}{D}=\frac{4.8}{\left(2400\times0.2+2400\times(0.4-0.2)\times\frac{1.4}{6.6}+1.6\right)}=\frac{4.8}{(4.7+1.0+1.6)}=0.66<\frac{3}{4}$$

a. Factored load and Fixed-End Moments (FEM's).

Factored dead load, $w_{df}=1.25\times(4.7+1.0+1.6)=9.1\text{ kN/m}^2$

Factored live load, $w_{lf}=1.5\times4.8=7.2\text{ kN/m}^2$

Total factored load $q_u=w_f=w_{df}+w_{lf}=16.3\text{ kN/m}^2$

FEM's for slab-beams $=m_{NF}q_u\ell_2\ell_1^{\,2}$ **PCA Notes on ACI 318-11 (Table A1)**

$=0.0840\times16.3\times6.6\times7.5^2=509.6$ kN.m (For Interior Span)

$=0.0841\times16.3\times6.6\times6.7^2=407.1$ kN.m (For Exterior Span)

b. Moment distribution.

Moment distribution computations are shown in Table 1. Counterclockwise rotational moments acting on the member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{u,midspan}=M_o-\frac{M_{uL}+M_{uR}}{2}$$

Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example.

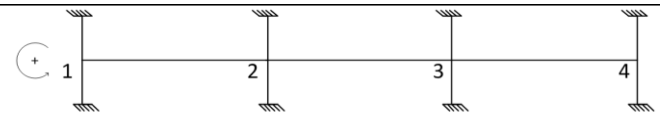
Positive moment in span 1-2:

$$M_u^+=\frac{(16.3\times6.6)\times6.7^2}{8}-\frac{(288.5+475.4)}{2}=221.7\text{ kN.m}$$

Positive moment span 2-3:

$$M_u^+=\frac{(16.3\times6.6)\times7.5^2}{8}-\frac{(504.9+504.9)}{2}=251.5\text{ kN.m}$$

Table 1 – Moment Distribution for Elastic Frame



Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.282	0.213	0.190	0.190	0.213	0.282
COF	0.510	0.510	0.500	0.500	0.510	0.510
FEM	407.10	-407.10	509.60	-509.60	407.10	-407.10
Dist	-114.80	-21.83	-19.48	19.48	21.83	114.80
CO	-11.13	-58.55	9.74	-9.74	58.55	11.13
Dist	3.14	10.40	9.27	-9.27	-10.40	-3.14
CO	5.30	1.60	-4.64	4.64	-1.60	-5.30
Dist	-1.50	0.65	0.58	-0.58	-0.65	1.50
CO	0.33	-0.77	-0.29	0.29	0.77	-0.33
Dist	-0.09	0.22	0.20	-0.20	-0.22	0.09
CO	0.11	-0.05	-0.10	0.10	0.05	-0.11
Dist	-0.03	0.03	0.03	-0.03	-0.03	0.03
CO	0.02	-0.02	-0.02	0.02	0.02	-0.02
Dist	0.00	0.01	0.01	-0.01	-0.01	0.00
CO	0.01	0.00	-0.01	0.01	0.00	-0.01
Dist	0.00	0.00	0.00	0.00	0.00	0.00
M, kN.m	288.50	-475.40	504.90	-504.90	475.40	-288.50
Midspan M, kN.m	221.73		251.52		221.73	

2.2.4. Design moments

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 13. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than $0.175\ell_1$ from the centers of supports. ***CSA A23.3-14 (13.8.5.1)***

$$\frac{600}{2} = 300 \text{ mm} < 0.175 \times 6700 = 1172.5 \text{ mm (use face of supporting location)}$$

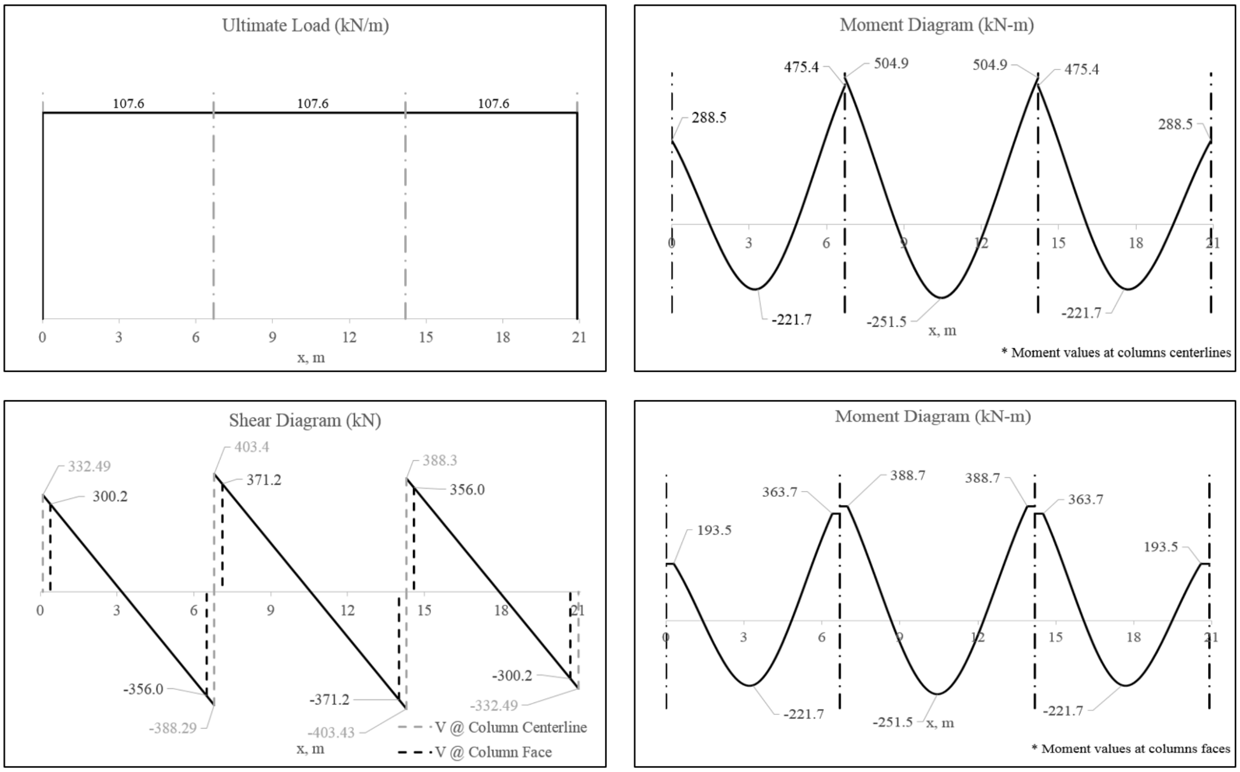


Figure 13 - Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load)

2.2.5. Distribution of design moments

After the negative and positive moments have been determined for the slab-beam strip, the CSA code permits the distribution of the moments at critical sections to the column strips, beams (if any), and middle strips in accordance with the DDM. ***CSA A23.3-14 (13.12)***

Beams shall be reinforced to resist the following fraction of the positive or interior negative factored moments determined by analysis or determined as specified in Clause 13.9.3. ***CSA A.23.3-14 (13.12.2.1)***

Portion of design moment resisted by beam:

$$\frac{\alpha_1}{0.3+\alpha_1}\left(1-\frac{l_2}{3l_1}\right)=\frac{2.12}{0.3+2.12}\left(1-\frac{6.6}{3\times 7.5}\right)=0.619 \text{ (for the interior span)}$$

$$\frac{\alpha_1}{0.3+\alpha_1}\left(1-\frac{l_2}{3l_1}\right)=\frac{2.12}{0.3+2.12}\left(1-\frac{6.6}{3\times 6.7}\right)=0.588 \text{ (for the exterior span)}$$

Beams shall be proportioned for 100% if the exterior negative moment. ***CSA A.23.3-14 (13.12.2.2)***

The slab shall be reinforced to resist the interior negative moments not resisted by the beams. This reinforcement shall be uniformly distributed over the width of the slab. ***CSA A.23.3-14 (13.12.4.1)***

The distribution factors for the remaining part of the column strip and middle strip are calculated as follows:

$$DF_{cs} = (1-0.619)\times\frac{1.9}{1.9+3.3} = 0.319$$
$$DF_{ms} = (1-0.619)\times\frac{3.3}{1.9+3.3} = 0.242$$
(interior span)

$$DF_{cs} = (1-0.588)\times\frac{1.9}{1.9+3.3} = 0.150$$
$$DF_{ms} = (1-0.588)\times\frac{3.3}{1.9+3.3} = 0.261$$
(M^+ section and M^- section at the interior support - exterior span)

Factored moments at critical sections are summarized in Table 2.

Table 2 - Lateral distribution of factored moments

		Factored Moments (kN.m)	Column Strip		Beam Strip		Two Half-Middle Strips*	
			Percent	Moment (kN.m)	Percent	Moment (kN.m)	Percent	Moment (kN.m)
End Span	Exterior Negative	193.5	100.0	193.5	0.0	0.0	0.0	0.0
	Positive	221.7	58.8	130.5	15.0	33.3	26.1	57.9
	Interior Negative	363.7	58.8	214.0	15.0	54.7	26.1	95.0
Interior Span	Negative	388.7	61.9	241.0	13.9	54.2	24.2	94.1
	Positive	251.5	61.9	155.7	13.9	35.0	24.2	60.8

* That portion of the factored moment not resisted by the column strip is assigned to the two half-middle strips

2.2.6. Flexural reinforcement requirements

a. Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the beam strip of interior span – negative location is provided below:

$M_f = 241$ kN.m

Beam strip width, b = 1400 mm

Use d = 400 – (25 + 16/2) = 367 mm

jd is assumed equal to *0.949d*. The assumption will be verified once the area of steel in finalized.

Assume $jd = 0.949 \times d = 348.3$ mm

$$A_s = \frac{M_f}{\phi_s f_y jd} = \frac{241 \times 10^6}{0.85 \times 400 \times 348.3} = 2035 \text{ mm}^2$$

$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.81 > 0.67$ **CSA A23.3-14 (10.1.7)**

$\beta_1 = 0.97 - 0.0025 f_c^{'} = 0.91 > 0.67$ **CSA A23.3-14 (10.1.7)**

Recalculate 'a' for the actual $A_s = 2035 \text{ mm}^2 \rightarrow a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85 \times 2035 \times 400}{0.65 \times 0.81 \times 25 \times 1400} = 37.4$ mm

$$c = \frac{a}{\beta_1} = \frac{37.4}{0.91} = 41.25 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$$\frac{c}{d} \le \frac{700}{700 + f_y}$$ **CSA A23.3-14 (10.5.2)**

$$\frac{41.25}{367} = 0.112 \le 0.640$$

$$jd = d - \frac{a}{2} = 0.949d$$

$$A_{s,\min} = \frac{0.2 \times \sqrt{f_c^{'}}}{f_y} \times b_t \times h = \frac{0.2\sqrt{25}}{400} \times 1400 \times 400 = 1400 \text{ mm}^2$$ **CSA A23.3-14 (10.5.1.2)**

$\therefore A_s = 2035 \text{ mm}^2$

Provide 11 – 25M bars with A_s = 2200 mm^2

The flexural reinforcement calculation for the column strip of exterior span – interior negative location is provided below:

$M_f = 54.7$ kN.m

Column strip width, b = (6600 /2) - 1400 = 1900 mm

Middle strip width, b = 6600 - 1900 - 1400 = 3300 mm

Use d = 200 – (25 + 16/2) = 167 mm

In this example, jd is assumed equal to $0.959d$. The assumption will be verified once the area of steel in finalized.

Assume $jd = 0.959 \times d = 160.2$ mm

$$A_s = \frac{M_f}{\phi_s f_y jd} = \frac{54.7 \times 10^6}{0.85 \times 400 \times 160.2} = 1004.3 \text{ mm}^2$$

$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.81 > 0.67$ **CSA A23.3-14 (10.1.7)**

Recalculate $'a'$ for the actual $A_s = 1004.6 \text{ mm}^2 \rightarrow a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85 \times 1004.6 \times 400}{0.65 \times 0.81 \times 25 \times 1900} = 13.62$ mm

$$c = \frac{a}{\beta_1} = \frac{13.62}{0.91} = 15 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$$\frac{c}{d} \leq \frac{700}{700 + f_y}$$ **CSA A23.3-14 (10.5.2)**

$$\frac{15}{167} = 0.09 \leq 0.64$$

$$jd = d - \frac{a}{2} = 0.959d$$

$A_{s,\min} = 0.002 \times 1900 \times 200 = 760 \text{ mm}^2 < 1004.3 \text{ mm}^2$ **CSA A23.3-14 (7.8.1)**

$\therefore A_s = 1004.3 \text{ mm}^2$

Maximum spacing: **CSA A23.3-14 (13.10.4)**

- Negative reinforcement in the band defined by b_b: $1.5h_s = 300 \text{ mm} > 250 \text{ mm} = 250$ mm
- Remaining negative moment reinforcement: $3h_s = 600 \text{ mm} > 500 \text{ mm} = 500$ mm

Provide 6 – 15M bars with A_s = 200 mm^2 and s = 1900/6 = 317 mm ≤ s_{max} = 500 mm

All the values on Table 3 are calculated based on the procedure outlined above.

Table 3 - Required Slab Reinforcement for Flexure (Elastic Frame Method (EFM))								
Span Location		**M_f (kN.m)**	**b (m)**	**d (mm)**	**A_s Req'd for flexure (mm^2)**	**Min A_s (mm^2)**	**Reinforcement Provided**	**A_s Prov. for flexure (mm^2)**
End Span								
Beam Strip	Exterior Negative	193.5	1.4	367	1616	1400	11 - 15M*	2200
	Positive	130.5	1.4	367	1075	1400	7 - 15M†	1400
	Interior Negative	214.0	1.4	367	1796	1400	11 - 15M**	2200
Column Strip	Exterior Negative	0.0	1.9	167	0	760	4 - 15M†	800
	Positive	33.3	1.9	167	601	760	4 - 15M†	800
	Interior Negative	54.7	1.9	167	1004	760	6 - 15M	1200
Middle Strip	Exterior Negative	0.0	3.3	167	0	1320	7 - 15M†	1400
	Positive	57.9	3.3	167	1045	1320	7 - 15M†	1800
	Interior Negative	95.0	3.3	167	1744	1320	9 - 15M	1800
Interior Span								
Beam Strip	Negative	241.0	1.4	367	2035	1400	11 - 15M**	2200
	Positive	155.7	1.4	367	1290	1400	7 - 15M†	1400
Column Strip	Negative	54.2	1.9	167	995	760	6 - 15M	1200
	Positive	35.0	1.9	167	633	760	4 - 15M†	800
Middle Strip	Negative	94.1	3.3	167	1727	1320	9 - 15M	1800
	Positive	60.8	3.3	167	1099	1320	7 - 15M†	1400

* The reinforcement is selected to meet CSA A23.3-14 provision 13.10.3.
** The reinforcement is selected to meet CSA A23.3-14 provision 13.11.2.7.
† Design governed by minimum reinforcement

b. Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of unbalanced moment given by γ_f shall be transferred by flexural reinforcement placed within a width b_b. ***CSA A23.3-14 (13.10.2)***

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_r$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ ***CSA A23.3-14 (13.10.2)***

Where

b_1 = Width width of the critical section for shear measured in the direction of the span for which moments are determined according to CSA A23.3-14, clause 13 (see Figure 14).

b_2 = Width of the critical section for shear measured in the direction perpendicular to b_1 according to CSA A23.3-14, clause 13 (see Figure 14).

b_b = Effective slab width $= c_2 + 3\times h_s$ ***CSA A23.3-14 (3.2)***

$$b_b = 400 + 3\times 200 = 1000 \text{ mm}$$

For Exterior Column

$$b_1 = 100 + 600 + \frac{367}{2} = 883.5 \text{ mm}$$

$$b_2 = 400 + 367 = 767 \text{ mm}$$

$$\gamma_f = \frac{1}{1 + (2/3) \times \sqrt{883.5/767}} = 0.583$$

For Interior Column

$$b_1 = 600 + 367 = 967 \text{ mm}$$

$$b_2 = 400 + 367 = 767 \text{ mm}$$

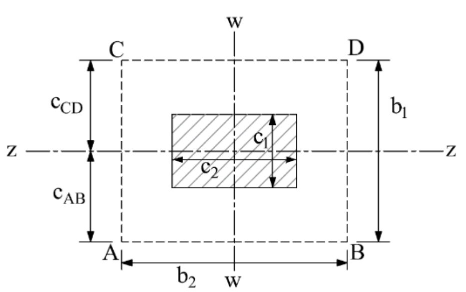
$$\gamma_f = \frac{1}{1 + (2/3) \times \sqrt{967/767}} = 0.572$$

Repeat the same procedure in section 2.2.6.a to calculate the additional reinforcement required for the unbalanced moment as shown in the following table:

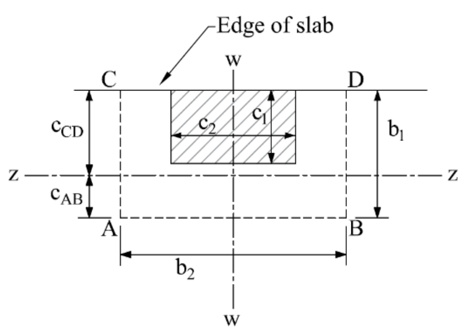
Table 4 - Additional Slab Reinforcement Required for Moment Transfer Between Slab and Column (EFM)

Span Location		M_u* (kN.m)	γ_f	$\gamma_f M_u$ (kN.m)	Effective slab width, b_b (mm)	d (mm)	A_s req'd within b_b (mm^2)	A_s prov. For flexure within b_b (mm^2)	Add'l Reinf.
End Span									
Column Strip	Exterior Negative	288.5	0.583	168	1000	367	1418	1800	-
	Interior Negative	29.5	0.572	16.9	1000	367	136	1800	-

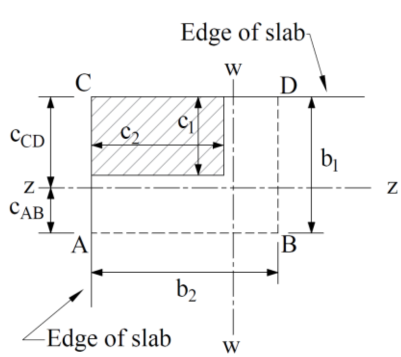
*M_u is taken at the centerline of the support in Elastic Frame Method solution.



Critical shear perimeter for interior column



Critical shear perimeter for exterior column



Critical shear perimeter for corner column

Figure 14 - Critical Shear Perimeters for Columns

c. Determine transverse reinforcement required for beam strip shear

The transverse reinforcement calculation for the beam strip of interior span is provided below.

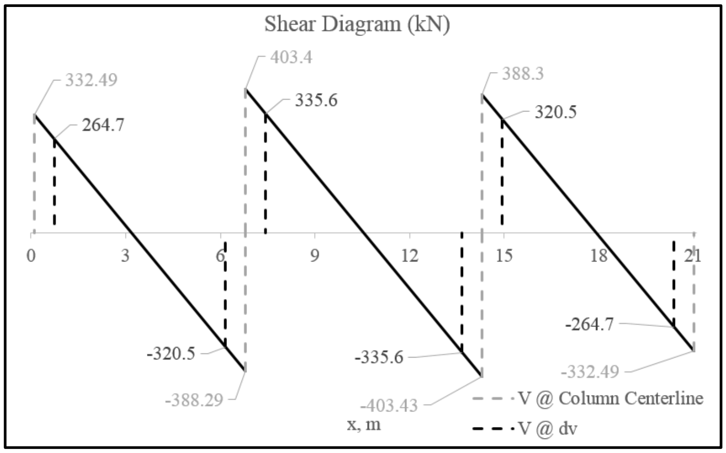


Figure 15 – Shear at critical sections (at distance d_v from the face of the column)

$$d_v = Max\ (0.9d, 0.72h) = Max\ (0.9 \times 367, 0.72 \times 400) = 330.3\,\text{mm}$$ **CSA A23.3-14 (3.2)**

The required shear at a distance d_v from the face of the supporting column $V_{f@dv}$ = 335.6 kN.

$$V_{r,\max} = 0.25 \times \phi_c \times f_c' \times b_w \times d_v$$ **CSA A23.3-14 (11.3.3)**

$$V_{r,\max} = 0.25 \times 0.65 \times 25 \times 1400 \times 330.3 / 1000 = 1879 \text{ kN}$$

$$V_{f@dv} = 335.6 \text{ kN} < V_{r,\max} = 1879 \text{ kN} \rightarrow \therefore \text{section is adequate}$$

$$V_c = \phi_c \lambda \beta \sqrt{f_c'} b_w d_v$$ **CSA A23.3-14 (Eq. 11.6)**

$$V_c = 0.65 \times 1.0 \times 0.21 \times \sqrt{25} \times 1400 \times 330.3 / 1{,}000 = 315.6 \text{ kN} < V_{f@dv} = 335.6 \text{ kN}$$

$\therefore$ Stirrups are required. (While this may cause concern during construction and complicate bar and concrete operations, it will be continued for illustration of the required calculations in this example. Ideally, a revised geometry, material strength, and loading should be considered to eliminate shear reinforcement. This reference example did not include detailed beam shear calculations to reveal the need for stirrups).

This beam is cast integrally with the slab where the overall depth (400 mm) is not greater than one-half the width of web (1400/2 = 700 mm) or 550 mm. thus, the value of β shall be taken as 0.21 and θ shall be taken as 42°. **CSA A23.3-14 (11.3.6.2(e))**

$$\sqrt{f_c'} = \sqrt{25} = 5 \text{ MPa} < 8 \text{ MPa}$$ **CSA A23.3-14 (11.3.4)**

The following shows how to calculate the distance from the column face beyond which transverse reinforcement is required:

$V_s = V_{f@dv} - V_c$ **CSA A23.3-14 (11.3.3)**

$V_s = 335.6 - 315.6 = 20 \text{ kN}$

$$\left(\frac{A_v}{s}\right)_{req} = \frac{V_{f@dv} - V_c}{\phi_s \times f_{yt} \times d_v \times \cot\theta}$$ **CSA A23.3-14 (11.3.5.1)**

$$\left(\frac{A_v}{s}\right)_{req} = \frac{20 \times 1000}{0.85 \times 400 \times 330.3 \times \cot 42^\circ} = 0.160 \text{ mm}^2/\text{mm}$$

$$\left(\frac{A_v}{s}\right)_{\min} = \frac{0.06 \times \sqrt{f_c'} \times b_w}{f_{yt}}$$ **CSA A23.3-14 (11.2.8.2)**

$$\left(\frac{A_v}{s}\right)_{\min} = \frac{0.06 \times \sqrt{25} \times 1400}{400} = 1.05 \text{ mm}^2/\text{mm (Governs)}$$

$$s_{req} = \frac{A_v}{\left(\frac{A_v}{s}\right)_{req}} = \frac{2 \times 100}{1.05} = 190.5 \text{ mm}$$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per **CSA A23.3-14 (11.3.8)**.

$0.125\lambda\phi_c f_c' b_w d_v > V_{f@dv}$ **CSA A23.3-14 (11.3.8.3)**

$$0.125\lambda\phi_c f_c' b_w d_v = 0.125 \times 1.0 \times 0.65 \times 25 \times 1400 \times 330.3 = 939.3 \text{ kN} > V_{f@dv} = 335.6 \text{ kN}$$

Therefore, maximum stirrup spacing shall be the smallest of $0.7d_v$ and 600 mm.

CSA A23.3-14 (11.3.8.1 & 11.3.8.3)

$$s_{\max} = \text{lesser of} \begin{bmatrix} 0.7d_v \\ 600 \text{ mm} \end{bmatrix} = \text{lesser of} \begin{bmatrix} 0.7 \times 330.3 \\ 600 \text{ mm} \end{bmatrix} = \text{lesser of} \begin{bmatrix} 231 \text{ mm} \\ 600 \text{ mm} \end{bmatrix} = 231 \text{ mm}$$

Since $s_{req} > s_{max} \rightarrow$ use $s_{req} = 190$ mm

Select $s_{provided}$ = 175 mm – 10M stirrups with first stirrup located at distance 76.2 mm (3 in.) from the column face.

The distance where the shear is zero is calculated as follows:

$$x = \frac{l}{V_{f,L} + V_{f,R}} \times V_{u,L} = \frac{7.5}{403.4 + 403.4} \times 403.4 = 3.75 \text{ m} = 3750 \text{ mm}$$

The distance at which no shear reinforcement is required is calculated as follows:

$$x_1 = x - \frac{x}{V_f} \times (0.85 \times V_c) = 3.75 - \frac{3.75}{403.4} \times (0.85 \times 315.6) = 1.256 \text{ m} = 1256 \text{ mm}$$

The following two provisions from CSA A23.3-14 explain the use of 85% of V_c:

The reudctions of shear resistance caused by terminating longitudinal reinforcement in flexural tension zones shall be taken into account. It can be assumed that the reductions in shear capacity occur over a length d_v centred upon the termination point. **CSA A23.3-14 (11.2.13.1)**

Note that if the factored shear resistance has been calculated using the simplified method of either **Clasue 11.3.6.2** or **Clause 11.3.6.3** then the calculated shear resistance within the length specified in **Clause 11.2.13.1** shall be reduced by 15% (85% of V_c as shown in the previous equation).

CSA A23.3-14 (11.2.13.2)

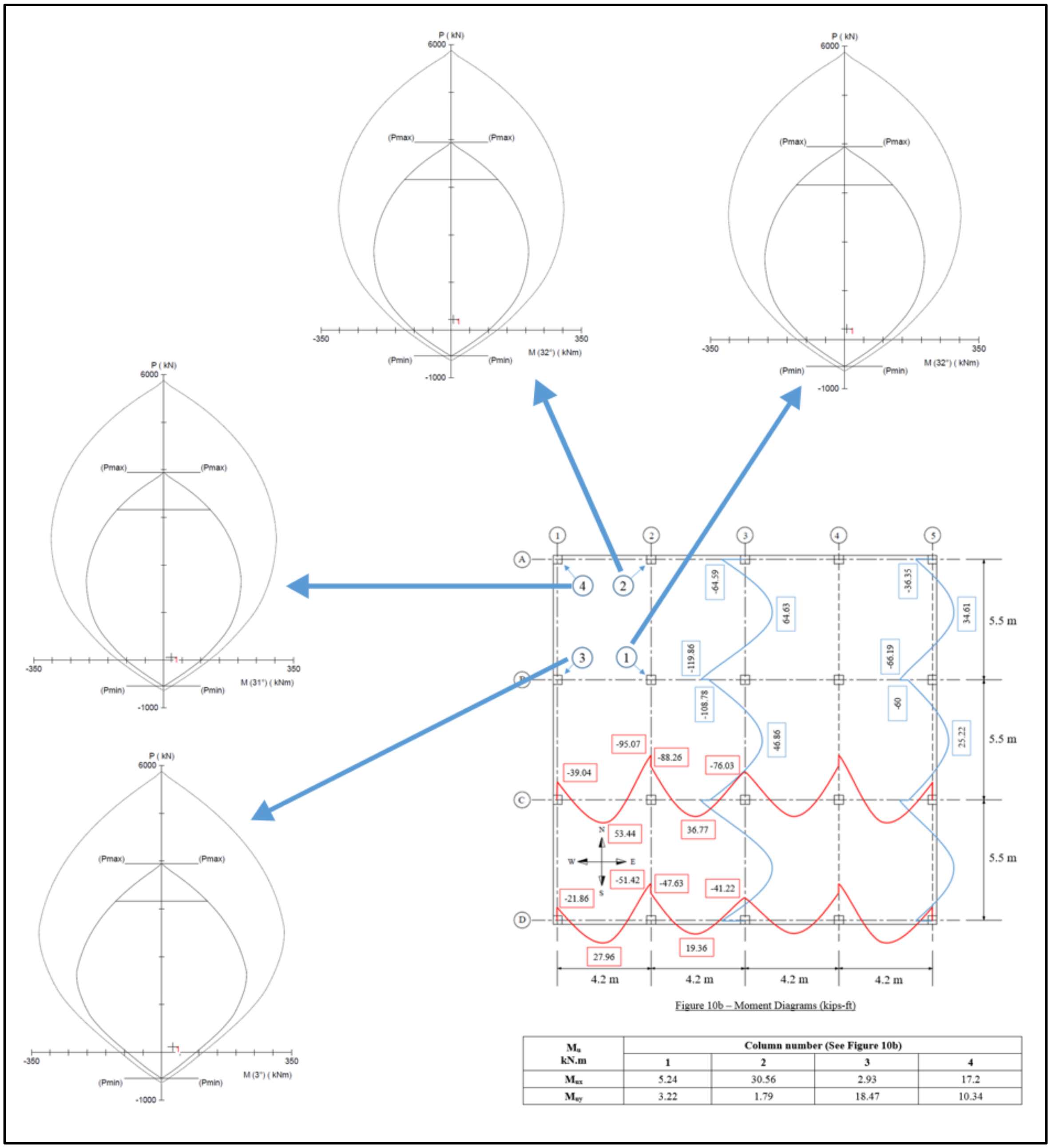
$$\#\,of\ \ stirrups = \frac{x_1 - \frac{c_1}{2} - 76.2\text{ mm}}{s_{provided}} + 1 = \frac{1256 - \frac{600}{2} - 76.2}{175} + 1 = 6.03 \rightarrow \text{use 7 stirrups}$$

All the values on Table 5 are calculated based on the procedure outlined above.

Table 5 - Required Beam Reinforcement for Shear					
Span Location	**$(A_v/s)_{min}$ mm²/mm**	**$(A_v/s)_{req}$ mm²/mm**	**s_{req} mm**	**s_{max} mm**	**Reinforcement Provided**
End Span					
Exterior	0.0	0.0	---	---	---
Interior	1.05	0.04	190.5	231.5	5 – 10M @ 185 mm
Interior Span					
Interior	1.05	0.16	190.5	231.2	7 – 10M @ 175 mm

2.2.7. Column design moments

The unbalanced moment from the slab-beams at the supports of the elastic frame are distributed to the support columns above and below the slab-beam in proportion to the relative stiffness of the support columns. Detailed calculations regarding this topic (including column design for axial load and biaxial moments) can be found in "Two-Way Flat Plate Concrete Slab Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website.



Mu kN.m	Column number (See Figure 10b)			
	1	2	3	4
Mux	5.24	30.56	2.93	17.2
Muy	3.22	1.79	18.47	10.34

Figure 16 - Sample Calculations of Column Design from "Two-Way Flat Plate Concrete Slab Floor Analysis and Design" Design Example

3. Two-Way Slab Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with CSA A23.3-14 clause 13.

3.1. One-Way (Beam action) Shear Strength for The Slab ***CSA A23.3-14 (13.3.6)***

The beam is designed to resist 100% of the one-way shear and the slab one-way shear strength need not to be checked. However, the following shows the calculations of the slab one-way shear strength for illustration purposes.

$$V_c = \phi_c \lambda \beta \sqrt{f_c^{'}} b_w d_v$$ ***CSA A23.3-14 (Eq. 11.5)***

λ = 1 for normal weight concrete

β = 0.21 for slabs with overall thickness not greater than 350 mm ***CSA A23.3-14 (11.3.6.2)***

$$d_v = \text{Max }(0.9 d_{slab}, 0.72 h_{slab})$$ ***CSA A23.3-14 (3.2)***

$$d_v = \text{Max }(0.9 \times 167, 0.72 \times 200) = \text{Max }(150, 144) = 150 \text{ mm}$$

$$\sqrt{f_c^{'}} = \sqrt{25} = 5 \text{ MPa} < 8 \text{ MPa}$$ ***CSA A23.3-14 (11.3.4)***

$$V_c = 0.65 \times 1 \times 0.21 \times \sqrt{25} \times 6600 \times \frac{150}{1000} = 533.4 \text{ kN}$$

3.2. Two-Way (Punching) Shear Strength — ***CSA A23.3-14 (13.3.2)***

Two-way shear is critical on a rectangular section located at $d_v/2$ away from the face of the column as shown in Figure 14.

a. Exterior column:

The factored shear force (V_f) in the critical section is computed as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section (*d/2* away from column face).

$$V_f = 332.5 - 16.3\left(0.8834 \times 0.767\right) = 321.4\,\text{kN}$$

The factored unbalanced moment used for shear transfer, M_{unb}, is computed as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

$$\text{M}_{\text{unb}} = \text{M}_{\text{u}} - \text{M}_f\left(\frac{b_1 - c_{AB} - c_1/2 - 100\text{ mm}}{1000\text{ mm}}\right)$$

$$\text{M}_{\text{unb}} = 288.5 - 321.4\left(\frac{883.5 - 308 - 600/2 - 100}{1000}\right) = 232.1\text{ kN.m}$$

For the exterior column in Figure 14, the location of the centroidal axis z-z is:

$$\text{c}_{\text{AB}} = e = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}} = \frac{2\times(883.5\times367\times883.5/2)}{2\times883.5\times367+767\times367} = 308\text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$\text{J}_c = 2\left(\frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + \left(b_1 d\right)\left(\frac{b_1}{2} - c_{AB}\right)^2\right) + b_2 d c_{AB}^2$$

$$\text{J}_c = 2\left(\frac{883.5\times367^3}{12} + \frac{367\times883.5^3}{12} + (883.5\times367)\left(\frac{883.5}{2} - 308\right)^2\right) + 767\times367\times(308)^2 = 87.77\times10^9\text{ mm}^4$$

$$\gamma_v = 1 - \gamma_f = 1 - 0.583 = 0.417$$ ***CSA A23.3-14 (Eq. 13.8)***

The length of the critical perimeter for the exterior column:

$$\text{b}_\text{o} = 2\times\left(600 + 100 + \frac{367}{2}\right) + \left(400 + 367\right) = 2534\,\text{mm}$$

The two-way shear stress (v_u) can then be calculated as:

$$v_f = \frac{V_f}{b_o \times d} + \frac{\gamma_v M_{unb} e}{J}$$ ***CSA A23.3-14 (Eq.13.9)***

$$v_f = \frac{321.4\times1000}{2534\times367} + \frac{0.417\times(288.5\times10^6)\times308}{87.77\times10^9}$$

$$v_f = 0.3456 + 0.3398 = 0.685\text{ MPa}$$

The factored resisiting shear stress, v_r shall be the smallest of : ***CSA A23.3-14 (13.3.4.1)***

a) $v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1.5}\right)0.19\times0.65\times\sqrt{25} = 1.441\text{ MPa}$

Where $\beta_c = c_1/c_2$ = 600/400 = 1.5

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{3\times367}{2534}+0.19\right)\times1\times0.65\times\sqrt{25} = 2.03\text{ MPa}$

Where α_s = 3 for edge columns

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{25} = 1.235\text{ MPa}$

v_c = min (1.441, 2.030, 1.235) = 1.235 MPa

CSA A23.3 requires multiplying the value of v_c by 1300/(1000+d) if the effective depth used in the two-way shear calculations exceeds 300 mm. ***CSA A23.3-14 (13.3.4.3)***

$$v_c = \left(\frac{1300}{1000+367}\right)\times1.235 = 1.174\text{ MPa}$$

Since ($v_r = 1.174\text{ MPa} \geq v_f = 0.685\text{ MPa}$) at the critical section, the slab has adequate two-way shear strength at this joint.

b. Interior column:

$$V_f = (403.4+284.3)-16.3\times(0.967\times0.767) = 779.6\text{ kN}$$

$$M_{unb} = (504.9-475.4)-779.6\times(0) = 29.5\text{ kN.m}$$

For the interior column in Figure 14, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{b_1}{2} = \frac{967}{2} = 483.5\text{ mm}$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left(\frac{b_1 d^3}{12}+\frac{d b_1^3}{12}+(b_1 d)\left(\frac{b_1}{2}-c_{AB}\right)^2\right)+2b_2 d c_{AB}^2$$

$$J_c = 2\left(\frac{967\times367^3}{12}+\frac{367\times967^3}{12}+(967\times367)\left(\frac{967}{2}-483.5\right)^2\right)+2\times767\times367\times(483.5)^2 = 194.9\times10^9\text{ mm}^4$$

$\gamma_v = 1-\gamma_f = 1-0.572 = 0.428$ ***CSA A23.3-14 (Eq. 13.8)***

The length of the critical perimeter for the interior column:

$$b_o = 2\times(600+367)+2\times(400+367) = 3468\text{ mm}$$

$v_f = \frac{V_f}{b_o\times d}+\frac{\gamma_v M_{unb} e}{J}$ ***CSA A23.3-14 (Eq.13.9)***

$$v_f = \frac{779.6\times1000}{3468\times367}+\frac{0.428\times(29.5\times10^6)\times483.5}{194.9\times10^9}$$

$$v_f = 0.613+0.031 = 0.644 \text{ MPa}$$

The factored resisiting shear stress, V_r shall be the smallest of : ***CSA A23.3-14 (13.3.4.1)***

a) $v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1.5}\right)0.19\times0.65\times\sqrt{25} = 1.441 \text{ MPa}$

b) $v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{4\times367}{3468}+0.19\right)\times1\times0.65\times\sqrt{25} = 1.993 \text{ MPa}$

c) $v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{25} = 1.235 \text{ MPa}$

v_c = min (1.441, 1.993, 1.235) = 1.235 MPa

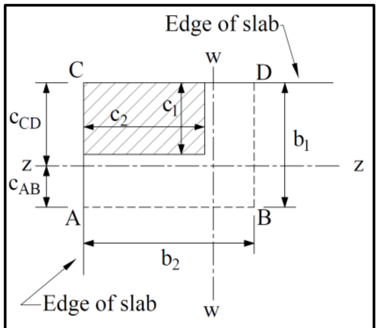
CSA A23.3 requires multiplying the value of v_c by 1300/(1000+d) if the effective depth used in the two-way shear calculations exceeds 300 mm. ***CSA A23.3-14 (13.3.4.3)***

$$v_c = \left(\frac{1300}{1000+367}\right)\times1.235 = 1.174 \text{ MPa}$$

Since ($v_r = 1.174 \text{ MPa} \geq v_f = 0.660 \text{ MPa}$) at the critical section, the slab has adequate two-way shear strength at this joint.

c. Corner column:

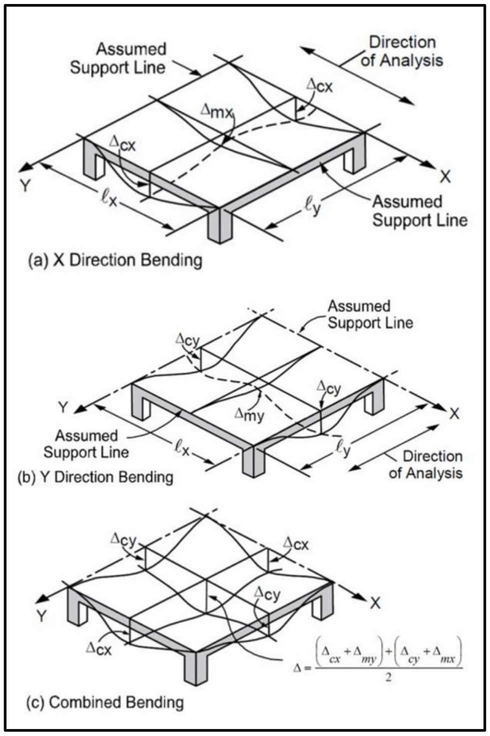
In this example, interior equivalent elastic frame strip was selected where it only have exterior and interior supports (no corner supports are included in this strip). Detailed calculations for two-way (punching) shear check around corner supports can be found in "Two-Way Flat Plate Concrete Slab Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website.



Critical shear perimeter for corner column

4. Two-Way Slab Deflection Control (Serviceability Requirements)

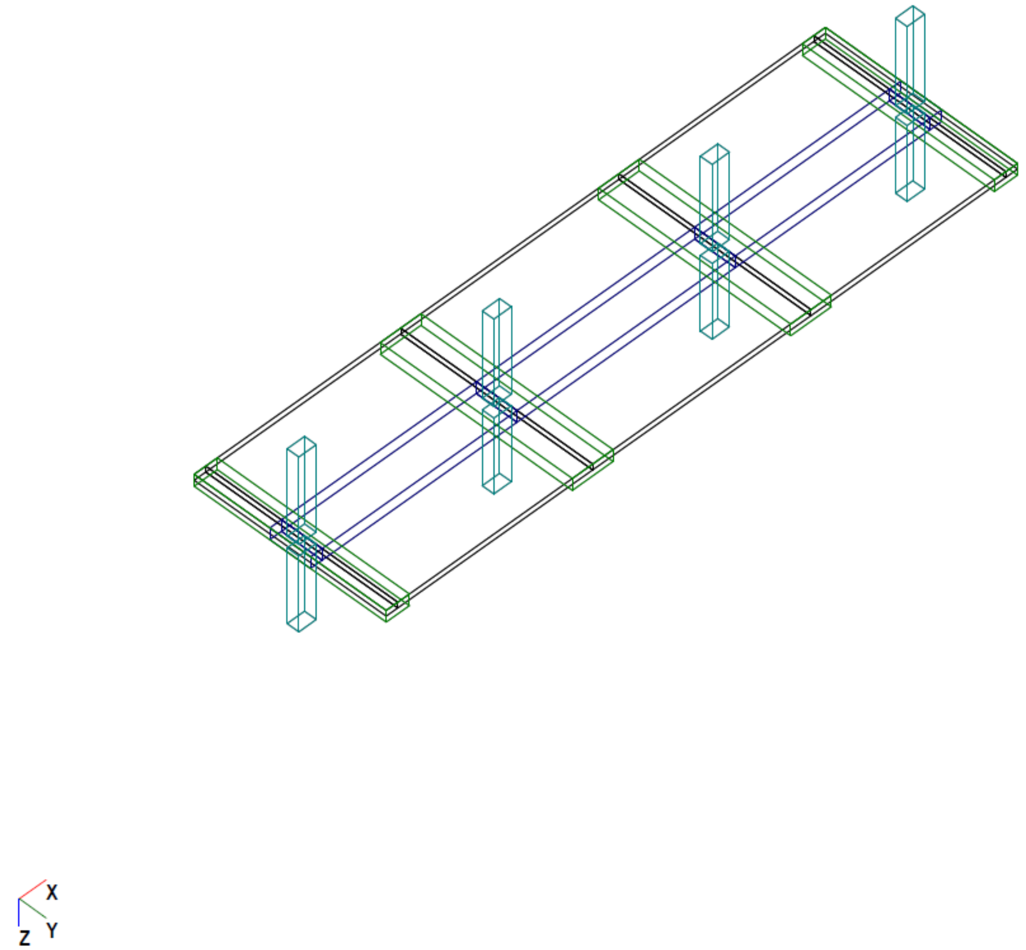
Since the slab thickness was selected based on the minimum slab thickness equations in CSA A23.3-14, the deflection calculations are not required. Detailed calculations of immediate and time-dependent deflections can be found in "Two-Way Concrete Slab on Beams Floor Analysis and Design (CSA A23.3-14)" example available in the design examples page in StructurePoint website.



5. spSlab Software Solution

spSlab program utilizes the Elastic (Equivalent) Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept.

spSlab Program models the equivalent elastic frame as a design strip. The design strip is, then, separated by spSlab into beam, column, and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for beam, column, and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for beam, column, and middle strips. The graphical and text results are provided below for both input and output of the spSlab model.



spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition - Section 5.7 - Example 4

Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 09/17/19

Time: 14:18:08

spSlab v5.50. Licensed to: StructurePoint. License ID:

File:

Project: CAC 4th Edition - Section 5.7 - Example 4

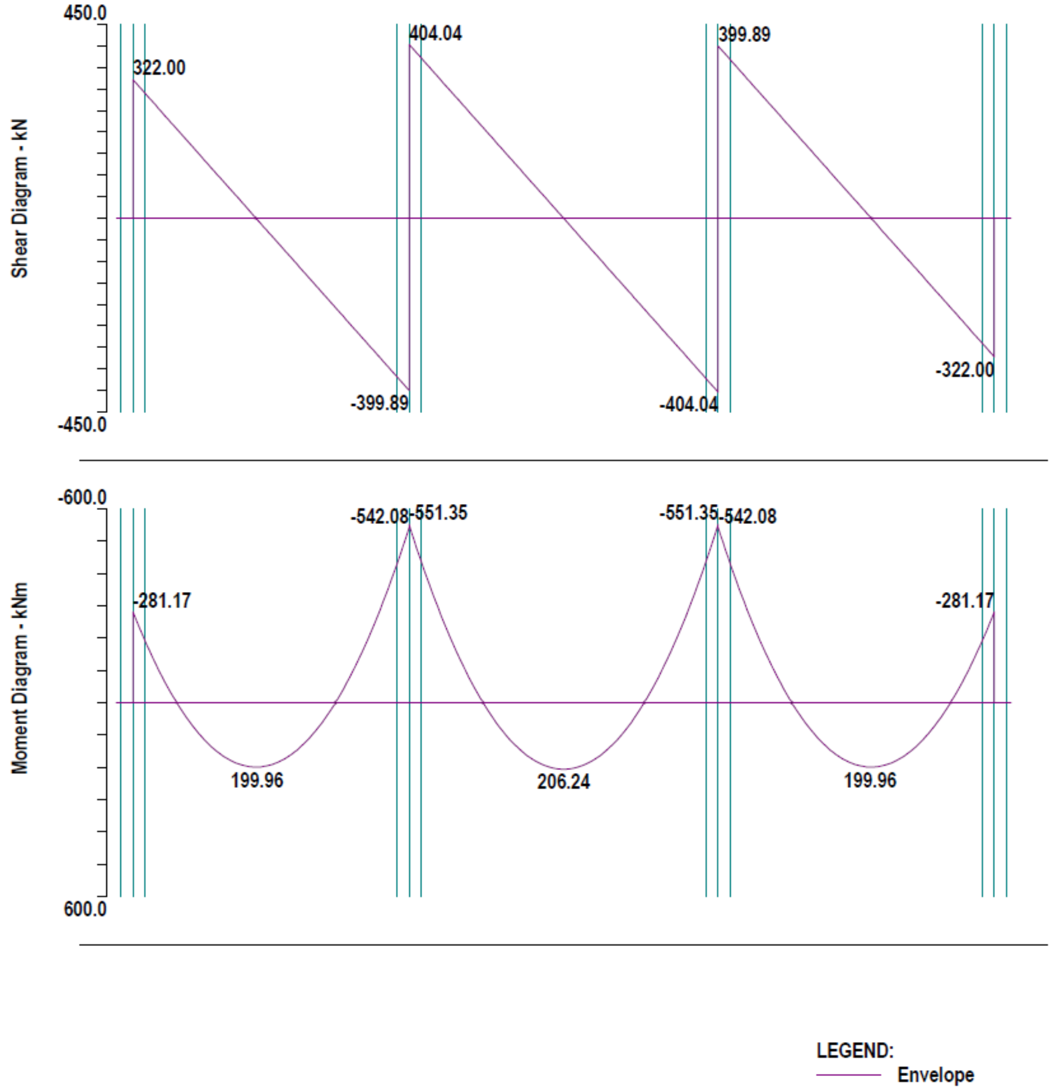
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 09/17/19

Time: 14:19:56



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File:

Project: CAC 4th Edition - Section 5.7 - Example 4

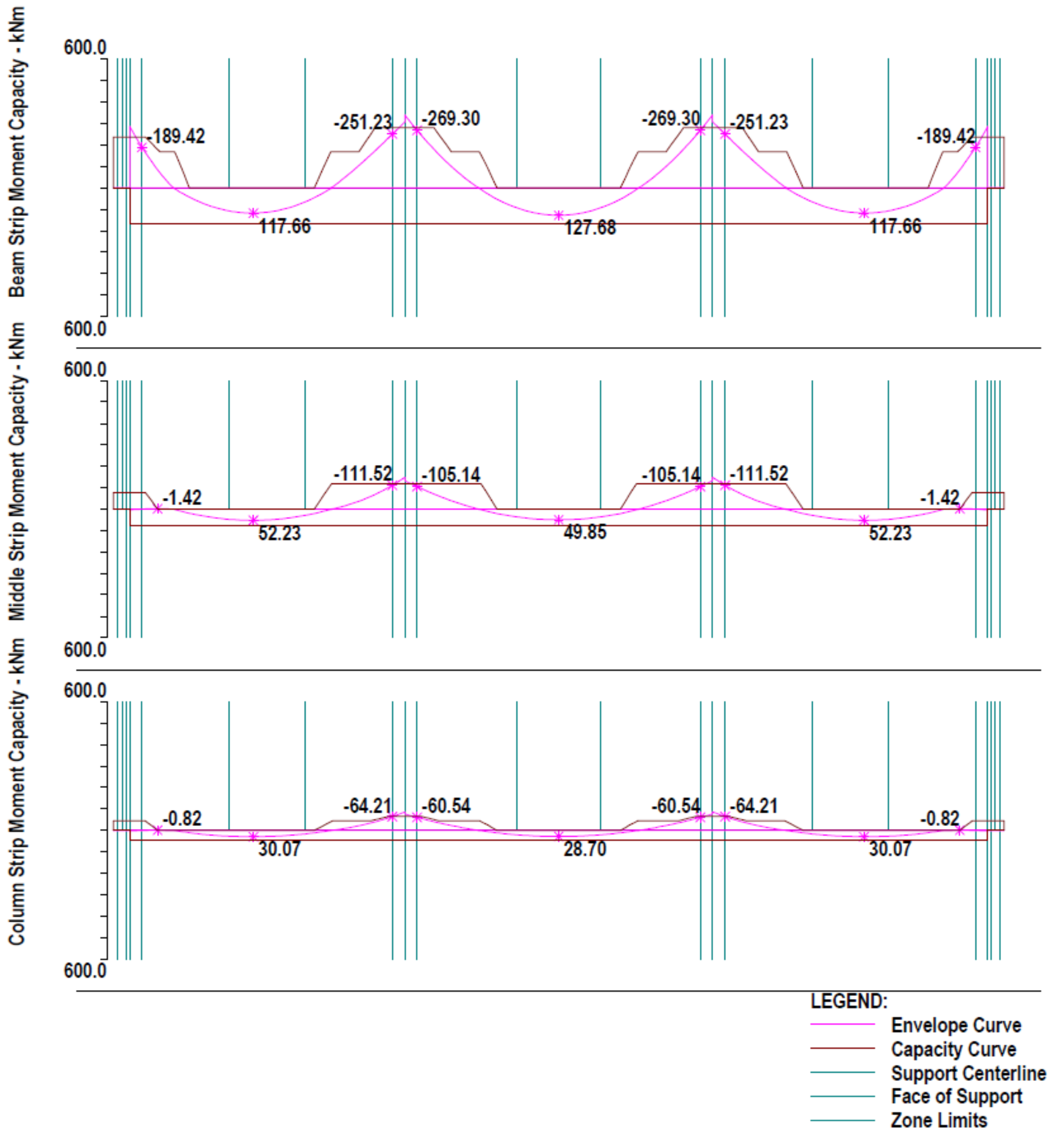
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 09/17/19

Time: 14:21:39



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File:

Project: CAC 4th Edition - Section 5.7 - Example 4

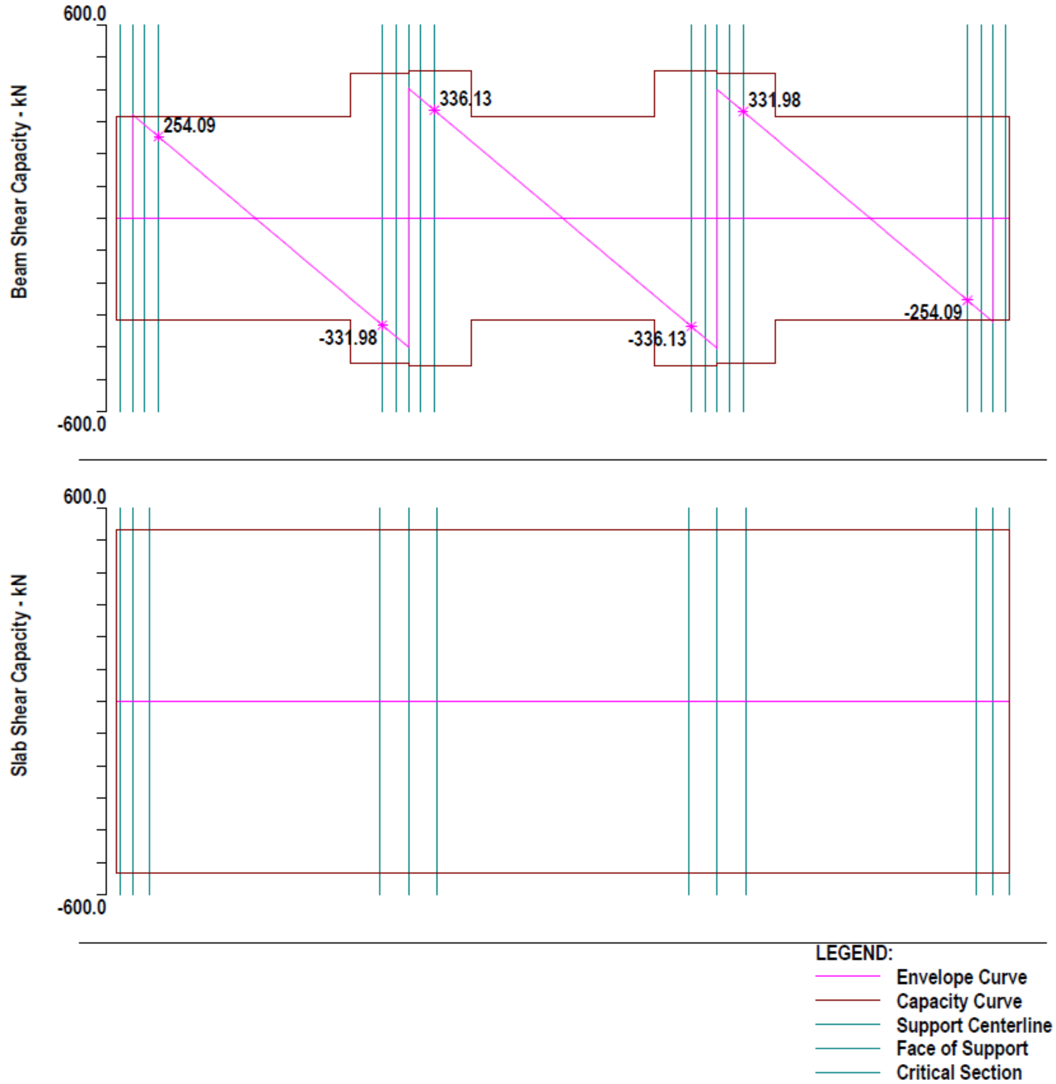
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 09/17/19

Time: 14:22:59



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File:

Project: CAC 4th Edition - Section 5.7 - Example 4

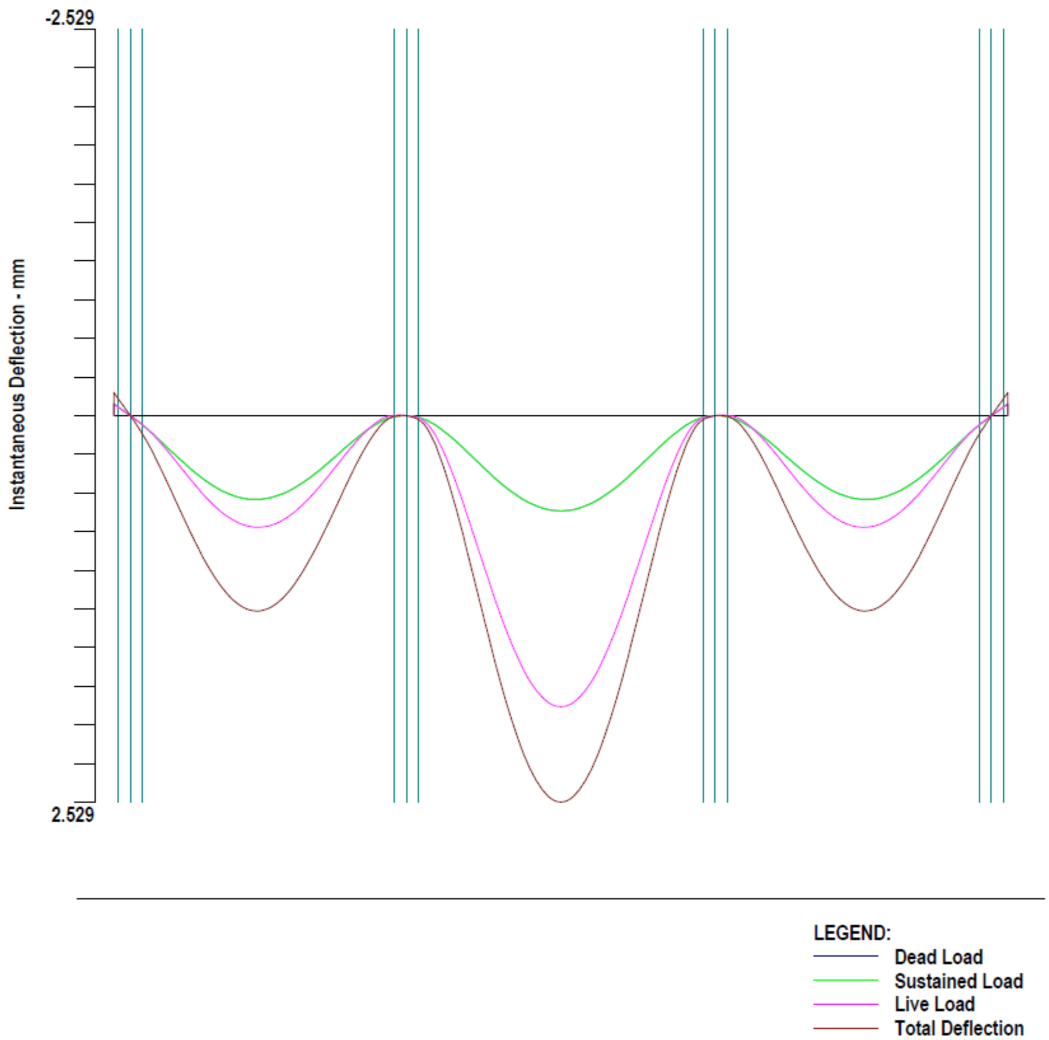
Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 09/17/19

Time: 14:24:00



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File:

Project: CAC 4th Edition - Section 5.7 - Example 4

Frame: Interior

Engineer: SP

Code: CSA A23.3-14

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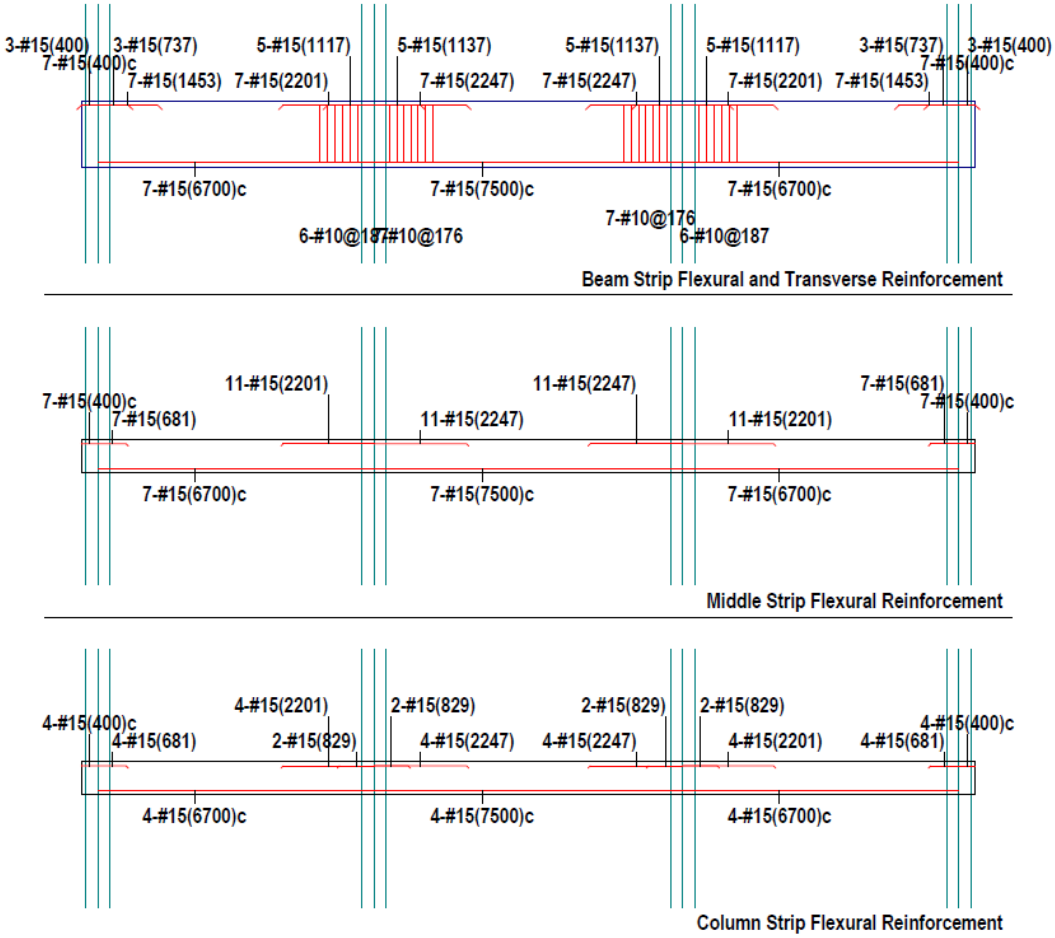
Time: 15:06:01



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File:

Project: CAC 4th Edition - Section 5.7 - Example 4

Frame: Interior

Engineer: SP

Code: CSA A23.3-14

Date: 09/17/19

Time: 15:07:19

spSlab v5.50
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems
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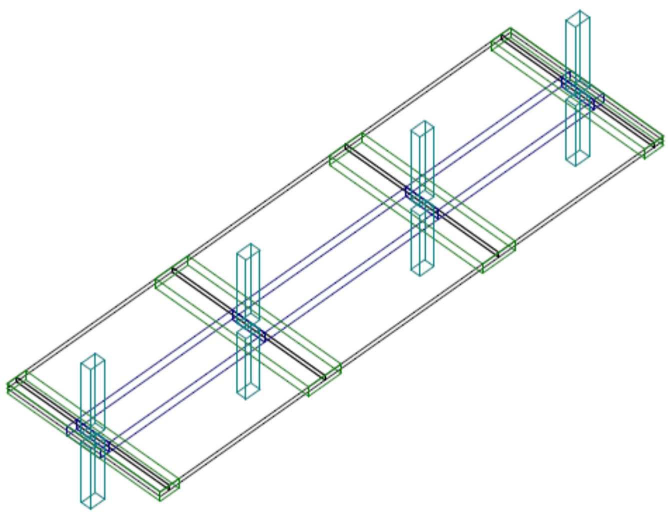

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Contents

1. Input Echo

1.1. General Information

File Name	C:\TSDA\Slab with Beams - CAC Example.slb
Project	CAC 4th Edition - Section 5.7 - Example 4
Frame	Interior
Engineer	SP
Code	CSA A23.3-14
Reinforcement Database	CSA G30.18
Mode	Design
Number of supports =	4 + Left cantilever + Right cantilever
Floor System	Two-Way

1.2. Solve Options

Live load pattern ratio = 0%
Minimum free edge distance for punching shear = 5 times slab effective depth.
Circular critical section around circular supports used (if possible).
Deflections are based on cracked section properties.
In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
Long-term deflections are calculated for load duration of 60 months.
0% of live load is sustained.
Compression reinforcement calculations NOT selected.
Default incremental rebar design selected.
Combined M-V-T reinforcement design NOT selected.
User-defined slab strip widths NOT selected.
User-defined distribution factors NOT selected.
One-way shear in drop panel NOT selected.
Distribution of shear to strips NOT selected.
Beam T-section design NOT selected.
Longitudinal beam contribution in negative reinforcement design over support NOT selected.
Transverse beam contribution in negative reinforcement design over support NOT selected.

1.3. Material Properties

1.3.1. Concrete: Slabs / Beams

w_c	2402.8	kg/m³
f'_c	25	MPa
E_c	24986	MPa
f_r	1.5	MPa
Precast concrete	No	

1.3.2. Concrete: Columns

w_c	2402.8	kg/m³
f'_c	25	MPa
E_c	24986	MPa
f_r	1.5	MPa
Precast concrete	No	

1.3.3. Reinforcing Steel

f_y	400 MPa
f_{yt}	400 MPa
E_s	200000 MPa
Epoxy coated bars	No

1.4. Reinforcement Database

Size	Db	Ab	Wb	Size	Db	Ab	Wb
	mm	mm²	kg/m		mm	mm²	kg/m
#10	11	100	1	#15	16	200	2
#20	20	300	2	#25	25	500	4
#30	30	700	5	#35	36	1000	8
#45	44	1500	12	#55	56	2500	20

1.5. Span Data

1.5.1. Slabs

Notes:
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Span	Loc	L1	t	wL	wR	L2L	L2R	H_{min}
		m	mm	m	m	m	m	mm
1	Int	0.400	200	3.300	3.300	6.600	6.600	--- LC *i
2	Int	6.700	200	3.300	3.300	6.600	6.600	163
3	Int	7.500	200	3.300	3.300	6.600	6.600	177
4	Int	6.700	200	3.300	3.300	6.600	6.600	163
5	Int	0.400	200	3.300	3.300	6.600	6.600	--- RC *i

1.5.2. Ribs and Longitudinal Beams

Notes:
*c - Deep beam. Additional design and bar detailing required.

Span	Ribs			Beams			
	b	h	Sp	b	h	Offset	
	mm	mm	mm	mm	mm	mm	
1	0	0	0	1400	400	0	*c
2	0	0	0	1400	400	0	
3	0	0	0	1400	400	0	
4	0	0	0	1400	400	0	
5	0	0	0	1400	400	0	*c

1.6. Support Data

1.6.1. Columns

Support	c1a	c2a	Ha	c1b	c2b	Hb	Red %
	mm	mm	m	mm	mm	m	
1	600	400	3.000	600	400	3.000	100
2	600	400	3.000	600	400	3.000	100
3	600	400	3.000	600	400	3.000	100
4	600	400	3.000	600	400	3.000	100

1.6.2. Transverse Beams

Supports	b	h	Ecc
	mm	mm	mm
1	800	400	0
2	1400	400	0
3	1400	400	0
4	800	400	0

1.6.3. Boundary Conditions

Support	Spring		Far End	
	K_z	K_{ry}	Above	Below
	kN/mm	kN-mm/rad		
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

1.7. Load Data

1.7.1. Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.250	1.500

1.7.2. Area Loads

Case/Patt	Span	Wa
		kN/m²
Dead	2	7.30
	3	7.30
	4	7.30
Live	2	4.80
	3	4.80
	4	4.80

1.8. Reinforcement Criteria

1.8.1. Slabs and Ribs

	Units	Top Bars		Bottom Bars	
		Min.	Max.	Min.	Max.
Bar Size		#15	#15	#15	#15
Bar spacing	mm	25	500	25	500
Reinf ratio	%	0.14	5.00	0.14	5.00
Clear Cover	mm	25		25	

There is NOT more than 300 mm of concrete below top bars.

1.8.2. Beams

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Bar Size		#15	#15	#15	#15	#10	#20
Bar spacing	mm	25	457	25	457	152	457
Reinf ratio	%	0.14	5.00	0.14	5.00		
Clear Cover	mm	25		25			
Layer dist.	mm	25		25			
No. of legs						2	6

	Units	Top Bars		Bottom Bars		Stirrups	
		Min.	Max.	Min.	Max.	Min.	Max.
Side cover	mm					38	
1st Stirrup	mm					76	

There is NOT more than 300 mm of concrete below top bars.

2. Design Results*

*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

2.1. Strip Widths and Distribution Factors

Notes:
*Used for bottom reinforcement. **Used for top reinforcement.

Span	Strip	Width Left **	Width Right **	Width Bottom *	Moment Factor Left **	Moment Factor Right **	Moment Factor Bottom *
		m	m	m	m	m	m
1	Column	1.90	1.90	1.90	0.000	0.000	0.152
	Middle	3.30	3.30	3.30	0.000	0.000	0.264
	Beam	1.40	1.40	1.40	1.000	1.000	0.584
2	Column	1.90	1.90	1.90	0.000	0.150	0.150
	Middle	3.30	3.30	3.30	0.000	0.261	0.261
	Beam	1.40	1.40	1.40	1.000	0.588	0.588
3	Column	1.90	1.90	1.90	0.139	0.139	0.139
	Middle	3.30	3.30	3.30	0.242	0.242	0.242
	Beam	1.40	1.40	1.40	0.619	0.619	0.619
4	Column	1.90	1.90	1.90	0.150	0.000	0.150
	Middle	3.30	3.30	3.30	0.261	0.000	0.261
	Beam	1.40	1.40	1.40	0.588	1.000	0.588
5	Column	1.90	1.90	1.90	0.000	0.000	0.152
	Middle	3.30	3.30	3.30	0.000	0.000	0.264
	Beam	1.40	1.40	1.40	1.000	1.000	0.584

2.2. Top Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.

Span	Strip	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
			m	kNm	m	mm²	mm²	mm²	mm	
1	Column	Left	1.90	0.00	0.000	760	7116	0	475	4-#15 *3
		Midspan	1.90	0.00	0.165	760	7116	0	475	4-#15 *3
		Right	1.90	0.00	0.330	760	7116	0	475	4-#15 *3
	Middle	Left	3.30	0.00	0.000	1320	12359	0	471	7-#15 *3
		Midspan	3.30	0.00	0.165	1320	12359	0	471	7-#15 *3
		Right	3.30	0.00	0.330	1320	12359	0	471	7-#15 *3
	Beam	Left	1.40	0.00	0.000	1400	11522	0	207	7-#15 *3
		Midspan	1.40	0.00	0.165	1400	11522	0	207	7-#15 *3
		Right	1.40	0.00	0.330	1400	11522	0	138	10-#15 *3
2	Column	Left	1.90	0.81	0.699	760	7116	14	475	4-#15 *3
		Midspan	1.90	0.00	3.350	0	7116	0	0	---

Span	Strip	Zone	Width	M_{max}	X_{max}	$A_{s,min}$	$A_{s,max}$	$A_{s,req}$	Sp_{Prov}	Bars
			m	kNm	m	mm²	mm²	mm²	mm	
		Right	1.90	64.21	6.400	760	7116	1188	317	6-#15
	Middle	Left	3.30	1.41	0.699	1320	12359	25	471	7-#15 *3
		Midspan	3.30	0.00	3.350	0	12359	0	0	---
		Right	3.30	111.52	6.400	1320	12359	2064	300	11-#15
	Beam	Left	1.40	189.42	0.300	1400	11522	1581	138	10-#15
		Midspan	1.40	0.00	3.350	0	11522	0	0	---
		Right	1.40	251.23	6.400	1400	11522	2127	113	12-#15
3	Column	Left	1.90	60.54	0.300	760	7116	1117	317	6-#15
		Midspan	1.90	0.00	3.750	0	7116	0	0	---
		Right	1.90	60.54	7.200	760	7116	1117	317	6-#15
	Middle	Left	3.30	105.14	0.300	1320	12359	1940	300	11-#15
		Midspan	3.30	0.00	3.750	0	12359	0	0	---
		Right	3.30	105.14	7.200	1320	12359	1940	300	11-#15
	Beam	Left	1.40	269.30	0.300	1400	11522	2290	113	12-#15
		Midspan	1.40	0.00	3.750	0	11522	0	0	---
		Right	1.40	269.30	7.200	1400	11522	2290	113	12-#15
4	Column	Left	1.90	64.21	0.300	760	7116	1188	317	6-#15
		Midspan	1.90	0.00	3.350	0	7116	0	0	---
		Right	1.90	0.81	6.001	760	7116	14	475	4-#15 *3
	Middle	Left	3.30	111.52	0.300	1320	12359	2064	300	11-#15
		Midspan	3.30	0.00	3.350	0	12359	0	0	---
		Right	3.30	1.41	6.001	1320	12359	25	471	7-#15 *3
	Beam	Left	1.40	251.23	0.300	1400	11522	2127	113	12-#15
		Midspan	1.40	0.00	3.350	0	11522	0	0	---
		Right	1.40	189.42	6.400	1400	11522	1581	138	10-#15
5	Column	Left	1.90	0.00	0.070	760	7116	0	475	4-#15 *3
		Midspan	1.90	0.00	0.235	760	7116	0	475	4-#15 *3
		Right	1.90	0.00	0.400	760	7116	0	475	4-#15 *3
	Middle	Left	3.30	0.00	0.070	1320	12359	0	471	7-#15 *3
		Midspan	3.30	0.00	0.235	1320	12359	0	471	7-#15 *3
		Right	3.30	0.00	0.400	1320	12359	0	471	7-#15 *3
	Beam	Left	1.40	0.00	0.070	1400	11522	0	138	10-#15 *3
		Midspan	1.40	0.00	0.235	1400	11522	0	207	7-#15 *3
		Right	1.40	0.00	0.400	1400	11522	0	207	7-#15 *3

2.3. Top Bar Details

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
			m		m		m		m		m
1	Column	---		---		4-#15	0.40	---		---	
	Middle	---		---		7-#15	0.40	---		---	

Span	Strip	Left Bars	Left Length (m)	Left Bars	Left Length (m)	Continuous Bars	Continuous Length (m)	Right Bars	Right Length (m)	Right Bars	Right Length (m)
	Beam	---		---		7-#15	0.40	3-#15	0.40	---	
2	Column	4-#15 *	0.68	---		---		4-#15	2.20	2-#15 *	0.83
	Middle	7-#15 *	0.68	---		---		11-#15	2.20	---	
	Beam	7-#15	1.45	3-#15 *	0.74	---		7-#15	2.20	5-#15 *	1.12
3	Column	4-#15	2.25	2-#15 *	0.83	---		4-#15	2.25	2-#15 *	0.83
	Middle	11-#15	2.25	---		---		11-#15	2.25	---	
	Beam	7-#15	2.25	5-#15 *	1.14	---		7-#15	2.25	5-#15 *	1.14
4	Column	4-#15	2.20	2-#15 *	0.83	---		4-#15 *	0.68	---	
	Middle	11-#15	2.20	---		---		7-#15 *	0.68	---	
	Beam	7-#15	2.20	5-#15 *	1.12	---		7-#15	1.45	3-#15 *	0.74
5	Column	---		---		4-#15	0.40	---		---	
	Middle	---		---		7-#15	0.40	---		---	
	Beam	3-#15	0.40	---		7-#15	0.40	---		---	

2.4. Top Bar Development Lengths

Span	Strip	Left Bars	Left DevLen (mm)	Left Bars	Left DevLen (mm)	Continuous Bars	Continuous DevLen (mm)	Right Bars	Right DevLen (mm)	Right Bars	Right DevLen (mm)
1	Column	---		---		4-#15	300.00	---		---	
	Middle	---		---		7-#15	300.00	---		---	
	Beam	---		---		7-#15	300.00	3-#15	300.00	---	
2	Column	4-#15	300.00	---		---		4-#15	443.99	2-#15	443.99
	Middle	7-#15	300.00	---		---		11-#15	420.62	---	
	Beam	7-#15	354.40	3-#15	354.40	---		7-#15	397.38	5-#15	397.38
3	Column	4-#15	417.33	2-#15	417.33	---		4-#15	417.33	2-#15	417.33
	Middle	11-#15	395.37	---		---		11-#15	395.37	---	
	Beam	7-#15	427.80	5-#15	427.80	---		7-#15	427.80	5-#15	427.80
4	Column	4-#15	443.99	2-#15	443.99	---		4-#15	300.00	---	
	Middle	11-#15	420.62	---		---		7-#15	300.00	---	
	Beam	7-#15	397.38	5-#15	397.38	---		7-#15	354.40	3-#15	354.40
5	Column	---		---		4-#15	300.00	---		---	
	Middle	---		---		7-#15	300.00	---		---	
	Beam	3-#15	300.00	---		7-#15	300.00	---		---	

2.5. Band Reinforcement at Supports

NOTES:
<C> Total Strip, <B> Banded Strip, <S> Remaining Strip

Support	Width <C> (mm)	Width <B> (mm)	Width <S> (mm)	A_s <C> (mm^2)	A_s <B> (mm^2)	A_s <S> (mm^2)	Bars <C>	Bars <B>	Bars <S>
1	1400	1000	400	2000	1600	400	10-#15	8-#15	2-#15
2	1400	1000	400	2400	1800	600	12-#15	9-#15	3-#15
3	1400	1000	400	2400	1800	600	12-#15	9-#15	3-#15
4	1400	1000	400	2000	1600	400	10-#15	8-#15	2-#15

2.6. Bottom Reinforcement

Notes:
*3 - Design governed by minimum reinforcement.

Span	Strip	Width m	M_{max} kNm	X_{max} m	$A_{s,min}$ mm²	$A_{s,max}$ mm²	$A_{s,req}$ mm²	Sp_{Prov} mm	Bars
1	Column	1.90	0.00	0.165	0	7116	0	0	---
	Middle	3.30	0.00	0.165	0	12359	0	0	---
	Beam	1.40	0.00	0.165	0	11522	0	0	---
2	Column	1.90	30.07	3.013	760	7116	542	475	4-#15 *3
	Middle	3.30	52.23	3.013	1320	12359	941	471	7-#15 *3
	Beam	1.40	117.66	3.013	1400	11522	966	207	7-#15 *3
3	Column	1.90	28.70	3.750	760	7116	516	475	4-#15 *3
	Middle	3.30	49.85	3.750	1320	12359	897	471	7-#15 *3
	Beam	1.40	127.68	3.750	1400	11522	1051	207	7-#15 *3
4	Column	1.90	30.07	3.687	760	7116	542	475	4-#15 *3
	Middle	3.30	52.23	3.687	1320	12359	941	471	7-#15 *3
	Beam	1.40	117.66	3.687	1400	11522	966	207	7-#15 *3
5	Column	1.90	0.00	0.235	0	7116	0	0	---
	Middle	3.30	0.00	0.235	0	12359	0	0	---
	Beam	1.40	0.00	0.235	0	11522	0	0	---

2.7. Bottom Bar Details

		Long Bars			Short Bars		
Span	Strip	Bars	Start m	Length m	Bars	Start m	Length m
1	Column	---			---		
	Middle	---			---		
	Beam	---			---		
2	Column	4-#15	0.00	6.70	---		
	Middle	7-#15	0.00	6.70	---		
	Beam	7-#15	0.00	6.70	---		
3	Column	4-#15	0.00	7.50	---		
	Middle	7-#15	0.00	7.50	---		
	Beam	7-#15	0.00	7.50	---		
4	Column	4-#15	0.00	6.70	---		
	Middle	7-#15	0.00	6.70	---		
	Beam	7-#15	0.00	6.70	---		
5	Column	---			---		
	Middle	---			---		
	Beam	---			---		

2.8. Bottom Bar Development Lengths

		Long Bars		Short Bars	
Span	Strip	Bars	DevLen mm	Bars	DevLen mm
1	Column	---		---	
	Middle	---		---	

Span	Strip	Long Bars		Short Bars	
		Bars	DevLen	Bars	DevLen
			mm		mm
	Beam	---		---	
2	Column	4-#15	303.53	---	
	Middle	7-#15	301.25	---	
	Beam	7-#15	309.53	---	
3	Column	4-#15	300.00	---	
	Middle	7-#15	300.00	---	
	Beam	7-#15	336.62	---	
4	Column	4-#15	303.53	---	
	Middle	7-#15	301.25	---	
	Beam	7-#15	309.53	---	
5	Column	---		---	
	Middle	---		---	
	Beam	---		---	

2.9. Longitudinal Beam Transverse Reinforcement Demand and Capacity

2.9.1. Section Properties

Span	d_v	$(A_v/s)_{min}$	ΦV_c	$V_{r,max}$
	mm	mm²/mm	kN	kN
1	330.3	1.050	315.60	1878.58
2	330.3	1.050	315.60	1878.58
3	330.3	1.050	315.60	1878.58
4	330.3	1.050	315.60	1878.58
5	330.3	1.050	315.60	1878.58

2.9.2. Beam Transverse Reinforcement Demand

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start	End	Required				Demand	
			X_u	V_u	Comb/Patt	A_v/s	A_v/s	
	m	m	m	kN		mm²/mm	mm²/mm	
1	0.000	0.024	0.000	0.00	U1/All	0.000	0.000	
2	0.376	1.407	0.630	254.09	U1/All	0.000	0.000	
	1.407	2.184	1.407	170.37	U1/All	0.000	0.000	
	2.184	2.961	2.184	86.64	U1/All	0.000	0.000	
	2.961	3.739	3.739	80.80	U1/All	0.000	0.000	
	3.739	4.516	4.516	164.53	U1/All	0.000	0.000	
	4.516	5.293	5.293	248.25	U1/All	0.000	0.000	
	5.293	6.324	6.070	331.98	U1/All	0.131	1.050	*8
3	0.376	1.522	0.630	336.13	U1/All	0.165	1.050	*8
	1.522	2.413	1.522	240.09	U1/All	0.000	0.000	
	2.413	3.304	2.413	144.06	U1/All	0.000	0.000	
	3.304	4.196	3.304	48.02	U1/All	0.000	0.000	
	4.196	5.087	5.087	144.06	U1/All	0.000	0.000	
	5.087	5.978	5.978	240.09	U1/All	0.000	0.000	
	5.978	7.124	6.870	336.13	U1/All	0.165	1.050	*8

Span	Start	End	Required X_u	V_u	Comb/Patt	A_v/s	Demand A_v/s	
	m	m	m	kN		mm²/mm	mm²/mm	
4	0.376	1.407	0.630	331.98	U1/All	0.131	1.050	*8
	1.407	2.184	1.407	248.25	U1/All	0.000	0.000	
	2.184	2.961	2.184	164.53	U1/All	0.000	0.000	
	2.961	3.739	2.961	80.80	U1/All	0.000	0.000	
	3.739	4.516	4.516	86.64	U1/All	0.000	0.000	
	4.516	5.293	5.293	170.37	U1/All	0.000	0.000	
	5.293	6.324	6.070	254.09	U1/All	0.000	0.000	
5	0.376	0.400	0.400	0.00	U1/All	0.000	0.000	

2.9.3. Beam Transverse Reinforcement Details

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#20	--- None ---
2	#10	<-- 4916 --> + 6 @ 187
3	#10	7 @ 176 + <-- 4457 --> + 7 @ 176
4	#10	6 @ 187 + <-- 4916 -->
5	#20	--- None ---

2.9.4. Beam Transverse Reinforcement Capacity

Notes:
*8 - Minimum transverse (stirrup) reinforcement governs.

Span	Start	End	Required X_u	V_u	Comb/Patt	A_v/s	Reqd/Min	Provided A_v	Sp	A_v/s	ΦV_n	
	m	m	m	kN		mm²/mm		mm²	mm	mm²/mm	kN	
1	0.000	0.400	0.000	0.00	U1/All	0.000	0.00	-----	-----	-----	315.60	
2	0.000	0.400	0.630	254.09	U1/All	-----	-----	-----	-----	-----	-----	
	0.400	0.630	0.630	254.09	U1/All	0.000	0.00	-----	-----	-----	315.60	
	0.630	5.293	0.630	254.09	U1/All	0.000	0.00	-----	-----	-----	315.60	
	5.293	6.000	6.000	324.47	U1/All	0.071	0.07	200.0	187	1.067	448.65	*8
	6.000	6.700	6.070	331.98	U1/All	-----	-----	-----	-----	-----	-----	
3	0.000	0.700	0.630	336.13	U1/All	-----	-----	-----	-----	-----	-----	
	0.700	1.522	0.700	328.62	U1/All	0.104	0.10	200.0	176	1.135	457.15	*8
	1.522	5.978	1.522	240.09	U1/All	0.000	0.00	-----	-----	-----	315.60	
	5.978	6.800	6.800	328.62	U1/All	0.104	0.10	200.0	176	1.135	457.15	*8
	6.800	7.500	6.870	336.13	U1/All	-----	-----	-----	-----	-----	-----	
4	0.000	0.700	0.630	331.98	U1/All	-----	-----	-----	-----	-----	-----	
	0.700	1.407	0.700	324.47	U1/All	0.071	0.07	200.0	187	1.067	448.65	*8
	1.407	6.070	6.070	254.09	U1/All	0.000	0.00	-----	-----	-----	315.60	
	6.070	6.300	6.070	254.09	U1/All	0.000	0.00	-----	-----	-----	315.60	
	6.300	6.700	6.070	254.09	U1/All	-----	-----	-----	-----	-----	-----	
5	0.000	0.400	0.400	0.00	U1/All	-----	-----	-----	-----	-----	-----	

2.10. Slab Shear Capacity

Span	b	d_v	β	V_{ratio}	ΦV_c	V_u	X_u
	mm	mm			kN	kN	m
1	5200	150	0.210	0.000	533.41	0.00	0.00
2	5200	150	0.210	0.000	533.41	0.00	5.85

Span	b	d_v	β	V_{ratio}	$ΦV_c$	V_u	X_u
	mm	mm			kN	kN	m
3	5200	150	0.210	0.000	533.41	0.00	0.85
4	5200	150	0.210	0.000	533.41	0.00	0.85
5	5200	150	0.210	0.000	533.41	0.00	0.00

2.11. Flexural Transfer of Negative Unbalanced Moment at Supports

Support	Width	Width-c	d	M_{unb}	Comb	Patt	γ_f	$A_{s,req}$	$A_{s,prov}$	Add Bars
	mm	mm	mm	kNm				mm²	mm²	
1	1000	1000	367	281.17	U1	All	0.583	1380	1600	---
2	1000	1000	367	9.27	U1	All	0.572	43	1800	---
3	1000	1000	367	9.27	U1	All	0.572	43	1800	---
4	1000	1000	367	281.17	U1	All	0.583	1380	1600	---

2.12. Punching Shear Around Columns

2.12.1. Critical Section Properties

Support	Type	b_1	b_2	b_0	d_{avg}	CG	$c_{(left)}$	$c_{(right)}$	A_c	J_c
		mm	mm	mm	mm	mm	mm	mm	mm²	mm⁴
1	Rect	883.5	767.0	2534.0	367.0	175.5	575.5	308.0	9.2998e+005	8.7765e+010
2	Rect	967.0	767.0	3468.0	367.0	0.0	483.5	483.5	1.2728e+006	1.9488e+011
3	Rect	967.0	767.0	3468.0	367.0	0.0	483.5	483.5	1.2728e+006	1.9488e+011
4	Rect	883.5	767.0	2534.0	367.0	-175.5	308.0	575.5	9.2998e+005	8.7765e+010

2.12.2. Punching Shear Results

Support	V_u	v_u	M_{unb}	Comb	Patt	γ_v	v_u	$ΦV_c$
	kN	N/mm²	kNm				N/mm²	N/mm²
1	315.95	0.340	225.74	U1	All	0.417	0.670	1.174
2	791.82	0.622	9.27	U1	All	0.428	0.632	1.174
3	791.82	0.622	-9.27	U1	All	0.428	0.632	1.174
4	315.95	0.340	-225.74	U1	All	0.417	0.670	1.174

2.13. Integrity Reinforcement at Supports

Notes:
Beams present. Integrity reinforcement may not be required.
The sum of bottom reinforcement crossing the perimeter of the support on all sides shall not be less than the below listed values.

Support	V_{se}	A_{sb}
	kN	mm²
1	263.64	1318 #
2	694.15	3471 #
3	694.15	3471 #
4	263.64	1318 #

2.14. Material TakeOff

2.14.1. Reinforcement in the Direction of Analysis

Top Bars	441.9 kg	<=>	20.36 kg/m	<=>	3.085 kg/m²
Bottom Bars	590.6 kg	<=>	27.22 kg/m	<=>	4.124 kg/m²
Stirrups	68.3 kg	<=>	3.15 kg/m	<=>	0.477 kg/m²
Total Steel	1100.8 kg	<=>	50.73 kg/m	<=>	7.686 kg/m²
Concrete	40.5 m³	<=>	1.87 m³/m	<=>	0.283 m³/m²

3. Deflection Results: Summary

3.1. Section Properties

3.1.1. Frame Section Properties

Notes:
M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Span	Zone	M_{+ve}			M_{-ve}		
		I_g	I_{cr}	M_{cr}	I_g	I_{cr}	M_{cr}
		mm^4	mm^4	kNm	mm^4	mm^4	kNm
1	Left	1.4573e+010	0	82.49	7.4667e+009	2.5181e+009	-56.00
	Midspan	1.4573e+010	0	82.49	7.4667e+009	2.8415e+009	-56.00
	Right	3.52e+010	0	264.00	3.52e+010	3.638e+009	-264.00
2	Left	3.52e+010	1.6218e+009	264.00	3.52e+010	3.638e+009	-264.00
	Midspan	1.4573e+010	1.6218e+009	82.49	7.4667e+009	0	-56.00
	Right	3.52e+010	1.6218e+009	264.00	3.52e+010	4.8384e+009	-264.00
3	Left	3.52e+010	1.6218e+009	264.00	3.52e+010	4.8384e+009	-264.00
	Midspan	1.4573e+010	1.6218e+009	82.49	7.4667e+009	0	-56.00
	Right	3.52e+010	1.6218e+009	264.00	3.52e+010	4.8384e+009	-264.00
4	Left	3.52e+010	1.6218e+009	264.00	3.52e+010	4.8384e+009	-264.00
	Midspan	1.4573e+010	1.6218e+009	82.49	7.4667e+009	0	-56.00
	Right	3.52e+010	1.6218e+009	264.00	3.52e+010	3.638e+009	-264.00
5	Left	3.52e+010	0	264.00	3.52e+010	3.638e+009	-264.00
	Midspan	1.4573e+010	0	82.49	7.4667e+009	2.8415e+009	-56.00
	Right	1.4573e+010	0	82.49	7.4667e+009	2.5181e+009	-56.00

3.1.2. Frame Effective Section Properties

Span	Zone	Weight	Load Level					
			Dead		Sustained		Dead+Live	
			M_{max}	I_e	M_{max}	I_e	M_{max}	I_e
			kNm	mm^4	kNm	mm^4	kNm	mm^4
1	Right	1.000	0.00	3.52e+010	0.00	3.52e+010	0.00	3.52e+010
	Span Avg	----	----	3.52e+010	----	3.52e+010	----	3.52e+010
2	Left	0.250	-125.73	3.52e+010	-125.73	3.52e+010	-208.40	3.52e+010
	Middle	0.500	89.42	1.1791e+010	89.42	1.1791e+010	148.21	3.8549e+009
	Right	0.250	-242.40	3.52e+010	-242.40	3.52e+010	-401.79	1.3451e+010
	Span Avg	----	----	2.3496e+010	----	2.3496e+010	----	1.409e+010
3	Left	0.250	-246.54	3.52e+010	-246.54	3.52e+010	-408.65	1.3024e+010
	Middle	0.500	92.22	1.0891e+010	92.22	1.0891e+010	152.86	3.6572e+009
	Right	0.250	-246.54	3.52e+010	-246.54	3.52e+010	-408.65	1.3024e+010
	Span Avg	----	----	2.3045e+010	----	2.3045e+010	----	8.3407e+009
4	Left	0.250	-242.40	3.52e+010	-242.40	3.52e+010	-401.79	1.3451e+010
	Middle	0.500	89.42	1.1791e+010	89.42	1.1791e+010	148.21	3.8549e+009
	Right	0.250	-125.73	3.52e+010	-125.73	3.52e+010	-208.40	3.52e+010
	Span Avg	----	----	2.3496e+010	----	2.3496e+010	----	1.409e+010
5	Left	1.000	0.00	3.52e+010	0.00	3.52e+010	0.00	3.52e+010
	Span Avg	----	----	3.52e+010	----	3.52e+010	----	3.52e+010

3.1.3. Strip Section Properties at Midspan

Notes:
Load distirubtion factor, LDL, averages moment distribution factors listed in Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Span	Column Strip			Middle Strip		
	I_g mm⁴	LDF	Ratio	I_g mm⁴	LDF	Ratio
1	1.09972e+010	0.868	1.150	2.2e+009	0.132	0.874
2	1.09972e+010	0.804	1.066	2.2e+009	0.196	1.298
3	1.09972e+010	0.758	1.005	2.2e+009	0.242	1.601
4	1.09972e+010	0.804	1.066	2.2e+009	0.196	1.298
5	1.09972e+010	0.868	1.150	2.2e+009	0.132	0.874

3.2. Instantaneous Deflections

3.2.1. Extreme Instantaneous Frame Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.08	---	-0.07	-0.07	-0.08	-0.15
		Loc	m	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	mm	0.55	---	0.73	0.73	0.55	1.28
		Loc	m	3.013	---	3.088	3.088	3.013	3.088
	Up	Def	mm	---	---	0.00	0.00	---	0.00
		Loc	m	---	---	6.475	6.475	---	6.625
3	Down	Def	mm	0.62	---	1.91	1.91	0.62	2.53
		Loc	m	3.750	---	3.750	3.750	3.750	3.750
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
4	Down	Def	mm	0.55	---	0.73	0.73	0.55	1.28
		Loc	m	3.687	---	3.612	3.612	3.687	3.612
	Up	Def	mm	---	---	0.00	0.00	---	0.00
		Loc	m	---	---	0.180	0.180	---	0.120
5	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.08	---	-0.07	-0.07	-0.08	-0.15
		Loc	m	0.400	---	0.400	0.400	0.400	0.400

3.2.2. Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.08	---	-0.07	-0.07	-0.08	-0.15
		Loc	m	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	mm	0.57	---	0.77	0.77	0.57	1.34
		Loc	m	3.088	---	3.088	3.088	3.088	3.088
	Up	Def	mm	---	---	0.00	0.00	---	0.00
		Loc	m	---	---	6.475	6.475	---	6.625
3	Down	Def	mm	0.63	---	1.91	1.91	0.63	2.54
		Loc	m	3.750	---	3.750	3.750	3.750	3.750
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
4	Down	Def	mm	0.57	---	0.77	0.77	0.57	1.34
		Loc	m	3.612	---	3.612	3.612	3.612	3.612

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
	Up	Def	mm	---	---	0.00	0.00	---	0.00
		Loc	m	---	---	0.180	0.180	---	0.120
5	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.08	---	-0.07	-0.07	-0.08	-0.15
		Loc	m	0.400	---	0.400	0.400	0.400	0.400

3.2.3. Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Span	Direction	Value	Units	Dead	Live			Total	
					Sustained	Unsustained	Total	Sustained	Dead+Live
1	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.08	---	-0.07	-0.07	-0.08	-0.15
		Loc	m	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	mm	0.66	---	0.90	0.90	0.66	1.56
		Loc	m	3.088	---	3.163	3.163	3.088	3.163
	Up	Def	mm	---	---	0.00	0.00	---	0.00
		Loc	m	---	---	6.550	6.550	---	6.625
3	Down	Def	mm	0.99	---	3.02	3.02	0.99	4.01
		Loc	m	3.750	---	3.750	3.750	3.750	3.750
	Up	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
4	Down	Def	mm	0.66	---	0.90	0.90	0.66	1.56
		Loc	m	3.612	---	3.537	3.537	3.612	3.537
	Up	Def	mm	---	---	0.00	0.00	---	0.00
		Loc	m	---	---	0.180	0.180	---	0.060
5	Down	Def	mm	---	---	---	---	---	---
		Loc	m	---	---	---	---	---	---
	Up	Def	mm	-0.08	---	-0.07	-0.07	-0.08	-0.15
		Loc	m	0.400	---	0.400	0.400	0.400	0.400

3.3. Long-term Deflections

3.3.1. Long-term Column Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span	Zone	M_{+ve}					M_{-ve}				
		$A_{s,top}$	b	d	Rho'	Lambda	$A_{s,bot}$	b	d	Rho'	Lambda
		mm^2	mm	mm	%		mm^2	mm	mm	%	
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.2. Long-term Middle Strip Deflection Factors

Notes:
Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Time dependant factor for sustained loads = 2.000

Span	Zone	M+ve					M-ve				
		$A_{s,top}$	b	d	Rho'	Lambda	$A_{s,bot}$	b	d	Rho'	Lambda
		mm^2	mm	mm	%		mm^2	mm	mm	%	
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

3.3.3. Extreme Long-term Column Strip Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	-0.15	-0.23	-0.23	-0.31
		Loc	m	0.000	0.000	0.000	0.000
2	Down	Def	mm	1.15	1.91	1.91	2.49
		Loc	m	3.088	3.088	3.088	3.088
	Up	Def	mm	---	0.00	0.00	0.00
		Loc	m	---	6.625	6.625	6.625
3	Down	Def	mm	1.25	3.17	3.17	3.79
		Loc	m	3.750	3.750	3.750	3.750
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
4	Down	Def	mm	1.15	1.91	1.91	2.49
		Loc	m	3.612	3.612	3.612	3.612
	Up	Def	mm	---	0.00	0.00	0.00
		Loc	m	---	0.060	0.060	0.060
5	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	-0.15	-0.23	-0.23	-0.31
		Loc	m	0.400	0.400	0.400	0.400

3.3.4. Extreme Long-term Middle Strip Deflections and Corresponding Locations

Notes:
Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
1	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	-0.15	-0.23	-0.23	-0.31
		Loc	m	0.000	0.000	0.000	0.000

Span	Direction	Value	Units	cs	cs+lu	cs+l	Total
2	Down	Def	mm	1.32	2.22	2.22	2.88
		Loc	m	3.088	3.088	3.088	3.088
	Up	Def	mm	---	0.00	0.00	0.00
		Loc	m	---	6.625	6.625	6.625
3	Down	Def	mm	1.98	5.00	5.00	5.99
		Loc	m	3.750	3.750	3.750	3.750
	Up	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
4	Down	Def	mm	1.32	2.22	2.22	2.88
		Loc	m	3.612	3.612	3.612	3.612
	Up	Def	mm	---	0.00	0.00	0.00
		Loc	m	---	0.060	0.060	0.060
5	Down	Def	mm	---	---	---	---
		Loc	m	---	---	---	---
	Up	Def	mm	-0.15	-0.23	-0.23	-0.31
		Loc	m	0.400	0.400	0.400	0.400

6. Summary and Comparison of Two-Way Slab Design Results

Table 6 – Summary of Flexural Design Moments				
		Reference (DDM)	Hand (EFM)	spSlab
Exterior Span				
Frame Strip	Exterior Negative	81	193.50	189.42
	Positive	299	221.70**	199.96* (191.46) **
	Interior Negative	354	363.70	426.96
Interior Span				
Frame Strip	Interior Negative	421	388.70	434.98
	Positive	227	251.50	206.24
* Maximum positive moment along exterior span (not at midspan)				
** Positive moment at the middle of the exterior span				

In Table 6, the negative moments are taken at the supports faces. Note that for the exterior span, the location of the maximum positive moment is not located at the mid span. The hand solution assumed that the maximum positive moment is located at the midspan for simplification. However, the spSlab program results provide the exact location of the maximum positive moment which is higher (199.96 kN.m) and will be used.

The reference used the Direct Design Method (DDM) to calculate the design moments, this method uses generic distribution factors for slabs with beams regardless of the geometric properties of the transverse and longitudinal beams. In spSlab and hand calculations, Elastic Frame Method (EFM) is being used, in this method, the exact geometric properties of the transverse and longitudinal beams are employed to perform the analysis and calculate the design moments.

In the hand calculations, the calculations of the moment distribution constants are approximated using the design aids tables for flat plates since tables for two-way slabs with beams are not available. On the other hand, spSlab calculates the exact values of these constants taking into account the effect of the longitudinal and transverse beams.

Table 7 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution			
		Hand (EFM)	**spSlab**
Exterior Span			
Beam Strip	Exterior Negative*	193.5	189.4
	Positive	130.5	117.7
	Interior Negative*	214.0	251.2
Column Strip	Exterior Negative*	0.0	0.0
	Positive	33.3	30.1
	Interior Negative*	54.7	64.2
Middle Strip	Exterior Negative*	0.0	0.0
	Positive	57.9	52.2
	Interior Negative*	95.0	111.5
Interior Span			
Beam Strip	Interior Negative*	241.0	269.3
	Positive	155.7	127.7
Column Strip	Interior Negative*	54.2	60.5
	Positive	35.0	28.7
Middle Strip	Interior Negative*	94.1	105.1
	Positive	60.8	49.9
* negative moments are taken at the faces of supports			

Table 8 - Comparison of Reinforcement Results with Hand and spSlab Solution							
Span Location		**Reinforcement Provided for Flexure**		**Additional Reinforcement Provided for Unbalanced Moment Transfer***		**Total Reinforcement Provided**	
		Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior Span							
Beam Strip	Exterior Negative	11- 15M	10- 15M	---	---	11- 15M	10- 15M
	Positive	7- 15M	7- 15M	n/a	n/a	7- 15M	7- 15M
	Interior Negative	11- 15M	12- 15M	---	---	11- 15M	12- 15M
Column Strip	Exterior Negative	4- 15M	4- 15M	n/a	n/a	4- 15M	4- 15M
	Positive	4- 15M	4- 15M	n/a	n/a	4- 15M	4- 15M
	Interior Negative	6- 15M	6- 15M	n/a	n/a	6- 15M	6- 15M
Middle Strip	Exterior Negative	7- 15M	7- 15M	n/a	n/a	7- 15M	7- 15M
	Positive	7- 15M	7- 15M	n/a	n/a	7- 15M	7- 15M
	Interior Negative	9- 15M	11- 15M	n/a	n/a	9- 15M	11- 15M
Interior Span							
Beam Strip	Negative	11 - 15M	12- 15M	---	---	11 - 15M	12- 15M
	Positive	7 - 15M	7- 15M	n/a	n/a	7 - 15M	7- 15M
Column Strip	Negative	6 - 15M	6- 15M	n/a	n/a	6 - 15M	6- 15M
	Positive	4 - 15M	4- 15M	n/a	n/a	4 - 15M	4- 15M
Middle Strip	Negative	9 - 15M	11- 15M	n/a	n/a	9 - 15M	11- 15M
	Positive	7 - 15M	7- 15M	n/a	n/a	7 - 15M	7- 15M

* In the EFM, the unbalanced moment (M_{sc}, M_{unb}) at the support centerline is used to determine the value of the additional reinforcement as compared with DDM using the moments at the face of support.

Table 9 - Comparison of Beam Shear Reinforcement Results		
Span Location	**Reinforcement Provided**	
	Hand	**spSlab**
End Span		
Exterior	---	---
Interior	5 – 10M @ 185 mm	6 – 10M @ 187 mm
Interior Span		
Interior	7 – 10M @ 175 mm	7 – 10M @ 176 mm

Table 10 - Comparison of Two-Way (Punching) Shear Check Results Using Hand and spSlab Solution

Support	b_1, mm		b_2, mm		b_o, mm		A_c, x 10^5 mm^2		V_u, kN		v_u, kN/mm^2	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	883.5	883.5	767	767	2534	2534	9.30	9.30	321.4	316.0	0.346	0.340
Interior	967	967	767	767	3468	3468	12.73	12.73	779.6	791.8	0.613	0.622

Support	c_{AB}, mm		J_c, x 10^9 mm^4		γ_v		M_{unb}, kN.m		v_u, MPa		ϕv_c, MPa	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	308	308	87.77	87.77	0.417	0.417	232.1	225.7	0.685	0.670	1.174	1.174
Interior	483.5	483.5	194.88	194.88	0.428	0.428	29.5	9.3	0.644	0.632	1.174	1.174

7. Comparison of Two-Way Slab Analysis and Design Methods

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in ***CSA A23.3-14 Clasues (13.8 and 13.9)*** for regular two-way slab systems. ***CSA A23.3-14 (13.5.1)***

Direct Design Method (DDM) is an approximate method and is applicable to flat plate concrete floor systems that meet the stringent requirements of ***CSA A23.3-14 (13.9.1)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Elastic Frame Method (EFM) has less stringent limitations compared to DDM. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming. StucturePoint's spSlab software program solution utilizes the EFM to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable CSA A23.3-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
13.8.1.1 13.9.1.1	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
13.8.1.1 13.9.1.1	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑	☑	
13.8.1.1 13.9.1.1	Column offset shall not exceed 20% of the span in direction of offset from either axis between centerlines of successive columns	☑	☑	
13.8.1.1 13.9.1.1	The reinforcement is placed in an orthogonal grid.	☑	☑	
13.9.1.2	Minimum of three continuous spans in each direction	☑		
13.9.1.3	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
13.9.1.4	All loads shall be due to gravity only	☑		
13.9.1.4	All loads shall be uniformly distributed over an entire panel (q_f)	☑		
13.9.1.4	Unfactored live load shall not exceed two times the unfactored dead load	☑		
13.10.6	Structural integrity steel detailing	☑	☑	☑
13.10.10	Openings in slab systems	☑	☑	☑
8.2	Concentrated loads	Not permitted	☑	☑
13.8.4.1	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
13.10.2*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
13.8.2	Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)	Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).